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INVESTIGATION OF LSMSA APPROACH IN LOCAL GEOID MODELING

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Abstract. This article presents the results of creating a gravimetric geoid model using a modified Stokes formula, implemented via the method of least squares with additive correction (LSMSA). Typically, the computation of geoid height is based on gravity measurements at points on or above the Earth's surface. However, practical limitations often restrict available gravity data to a spherical cap, leading to truncation errors. To mitigate these errors, it is essential to integrate terrestrial data with a global geopotential model (GGM). For an optimal gravimetric geoid definition, the GGM must be selected to best match the local gravity field. This selection is accomplished by comparing geodetic quantities derived from the GGM — such as potential, geoid heights, deflection of the vertical, gravity, and anomaly values — with those obtained from GNSS/levelling and terrestrial gravity measurements. This comparison reduces the impact of assumptions and approximations inherent in the Stokes method, thereby enhancing the accuracy of geoid height determination. To develop a local geoid using heterogeneous data sources (including gravity anomalies, digital high relief models, global geopotential models, and GNSS/levelling data) Least squares modification of Stokes formula with additive correction (LSMSA) method was chosen. This method is one of the simplest and most practical approaches, which has been successfully employed in regional geoid modelling. The LSMSA approach facilitates optimal integration of terrestrial and global data, contributing to more accurate and reliable geoid modelling.

Keywords: least squares modifications of Stokes formula, GGM, geoid, GNSS/leveling, accuracy assessment, standard deviation

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ЖЕРГІЛІКТІ ГЕОИДТЫ МОДЕЛЬДЕУДЕГІ LSMSA ТӘСІЛІН ЗЕРТТЕУ

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Аннотация. Мақалада геоидтың гравиметриялық моделін жасау нәтижелері Стокс формуласының модификацияларын аддитивті түзетумен ең кіші квадраттар әдісімен (LSMSA) қолдану арқылы ұсынылған. Әдетте, геоид биіктігін есептеу Жер бетінде немесе оның үстінде жер гравитациясын өлшеулер негізінде жүзеге асырылады. Алайда, тәжірибеде тек сфералық қалпақпен шектелген гравиметриялық деректер ғана қол жетімді, бұл кесу қатесіне әкеледі. Осы қатені азайту үшін жердегі деректерді ғаламдық гравитациялық потенциал моделімен (ГГМ) біріктіру қажет. Геоидтың оңтайлы гравиметриялық анықтамасына жету үшін жергілікті гравитациялық өріске ең жақсы сәйкес келетін ГГМ таңдау керек. Бұл ГГМ негізінде алынған геодезиялық шамаларды (мысалы, потенциал, геоид биіктіктері, тік компоненттердің ауытқуы, ауырлық күші және аномалия мәндері) ГНСС/Нивелирлеу және жердегі гравиметриялық өлшеулер негізінде алынған деректермен салыстыру арқылы жүзеге асырылады. Бұл Стокс әдісіне тән болжамдар мен жуықтаулардың әсерін азайтып, геоид биіктігін анықтаудың дәлдігін арттыруға мүмкіндік береді.

Әртүрлі деректерді (гравитациялық аномалиялар, жоғары цифрлық рельеф модельдері, ғаламдық гравитациялық потенциал модельдері, сондай-ақ ГНСС және нивелирлеу деректері) қолдана отырып, жергілікті геоидты әзірлеуде Стокс формуласының аддитивті түзетумен ең кіші квадраттар әдісімен модификациясын (LSMSA) пайдалану шешілді, бұл аймақтық геоид модельдерін анықтауда табысты қолданылатын ең қарапайым және практикалық тәсілдердің бірі болып табылады. Бұл тәсіл жергілікті және ғаламдық деректерді оңтайлы біріктіруді қамтамасыз етеді, бұл геоидты дәл және сенімді модельдеуге ықпал етеді.

Түйін сөздер: ең кіші квадраттар әдісін қолдану арқылы Стокс формуласының модификациясы, ГГМ, геоид, ГНСС/Нивелирлеу, дәлдікті бағалау, стандартты ауытқу

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ИССЛЕДОВАНИЕ ПОДХОДА LSMSA В МОДЕЛИРОВАНИИ ЛОКАЛЬНОГО ГЕОИДА

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Аннотация. В статье представлены результаты создания гравиметрической модели геоида с использованием модификаций формулы Стокса методом наименьших квадратов с аддитивной коррекцией (LSMSA). Обычно вычисление высота геоида осуществляется на основе измерений земной гравитации в точках на поверхности Земли или над ней. Однако на практике доступны лишь гравиметрические данные, ограниченные сферической шапкой, что приводит к ошибке усечения. Для уменьшения этой ошибки необходимо объединить наземные данные с глобальной геопотенциальной моделью (ГГМ). Для достижения оптимального гравиметрического определения геоида необходимо выбрать ГГМ, которая наилучшим образом соответствует местному гравитационному полю. Это достигается путем сравнения геодезических величин, полученных на основе ГГМ (таких как потенциал, высоты геоида, отклонение вертикальных компонентов, сила тяжести и значения аномалий) с данными, полученными на основе ГНСС/Нивелирования и наземных гравиметрических измерений. Это позволяет уменьшить влияние допущений и аппроксимаций, присущих методу Стокса, и повысить точность определения высоты геоида. При разработке локального геоида с применением разнородных данных (гравитационные аномалии, цифровые модели рельефа высокого, глобальные геопотенциальные модели, а также данные ГНСС и нивелирования) было решено использовать модификацию формулы Стокса методом наименьших квадратов с аддитивной коррекцией (LSMSA), которая представляет собой один из наиболее простых и практичных подходов, который успешно используется для определения региональных моделей геоида. Этот подход обеспечивает оптимальное объединение наземных и глобальных данных, что способствует более точному и надежному моделированию геоида.

Ключевые слова: модификаций формулы Стокса методом наименьших квадратов, ГГМ, геоид, ГНСС/Нивелирование, оценка точности, среднеквадратичное отклонение

Introduction

There are numerous methodologies for determining the regional geoid, each employing distinct techniques, which complicates the task of identifying the optimal method for a given situation. The fundamental approaches for calculating the geoid model frequently involve various modifications of the Stokes formula to enhance accuracy and adapt to specific conditions. The Stokes formula enables the determination of geoid height (N) from gravity data:

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} S(\psi)\Delta g d\sigma \tag{1}$$

where - is the average radius of the Earth, - geocentric angle, - gravitational anomaly, - infinitesimal surface element of integration over the unit sphere , - normal gravity on the reference ellipsoid, - the Stokes function. The Stokes function S(ψ) can be expressed in terms of Legendre polynomials $P_n(\cos\psi)$ using their orthogonality properties on the sphere:

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi) \tag{2}$$

The application of Stokes' theory to calculate height anomalies relative to the reference ellipsoid encounters significant challenges:

- For the required integration, gravity anomalies must be known across the entire Earth's surface. However, approximately one-third of the Earth's surface is covered by seas and oceans. The measurement of gravity anomalies over the oceans became feasible only in the twentieth century.

- The application of the Stokes formula assumes that all masses are located below the geoid. However, gravity anomaly measurements are conducted on the Earth's physical surface, which does not coincide with the level surface. To address this discrepancy, corrections are incorporated into the measured values to effectively reposition all topographic masses below sea level without altering the geoid surface. This ensures that gravity is related to the level surface. This issue, extensively discussed in the scientific literature, is known as the Earth regularization problem (Kaplan et al., 2017).

To date, one of the most prevalent methods based on the Stokes formula is the Least Squares Modification of Stokes Formula with Additive Correction (LSMSA), also known as the Royal Institute of Technology (KTH) method, developed by Professor Sjoberg. This method has gained widespread acceptance due to its effectiveness in improving the accuracy of geoid height determinations.

Materials and research methods

Least Squares Modification of Stokes Formula (LSMS) is one of the simplest and most practical approaches that has been successfully used to determine regional geoid models in various fields. The LSMS method integrates various heterogeneous data such as gravity anomalies, digital high relief models, global geopotential models, and GNSS and levelling data into a single least squares-based model. This approach ensures optimal data integration, which contributes to more accurate and reliable geoid modelling.

The LSMSA method, also known as the Royal Institute of Technology (KTH) approach to accurate geoid determination, has been developed by Professor Lars Sjöberg since 1984 (Goyal et al., 2017; Sjöberg, 1984; Sjöberg, 1991; Sjöberg, 2003). The theoretical and practical aspects of this method were finalised in 2006.

This method has been successfully applied in defining various regional geoid models. In history, this method was applied in the development of the Latvian geoid model. The RMS error was 7.5 cm in the Riga area using gravimetric data and a model of the Earth's gravity field (Sjöberg, 2003). In another study, the LSMSA method was used to determine the geoid in Poland, also applying LSMSA with additive corrections to compute a new gravimetric geoid model (Janpaule et al., 2014). In addition, the LSMSA method was used in a study to determine the geoid for the territory of Iran, although the main technique mentioned was Molodensky's method [(Kuczynska-Siehien, 2016). Also, the effectiveness of the method was demonstrated in a study on the geoid of South Korea, which showed a significant improvement in accuracy when new land gravity and GNSS/levelling data were included (Abdollahzadeh et al., 2011). The LSMSA method has been employed for local geoid determination by integrating spherical harmonic coefficients, gravity anomalies, and topographic data to achieve high accuracy, this method has been successfully applied in various regions, including Ethiopia (Bae et al., 2012), Sweden, the Baltic States (Bae et al., 2012), and Iran (Ellmann, 2004).

The method has also performed well in areas with very complex topography and in developing regions where gravity anomaly data are limited. This demonstrates the versatility and adaptability of this method to complex conditions. The modified Stokes method provides the approximate height of the geoid :

$$\tilde{N} = \frac{c}{2\pi}\iint_{\sigma_0} S^L(\psi)\Delta g d\sigma + c\sum_{n=2}^{M} b_n \Delta g_n^{GGM} \qquad (3)$$

where $c = \frac{R}{2\gamma}$

$S^L(\psi)$ – modified Stokes function;

b_n – modification parameters;

L – degree of modification of the core.

Using data error estimates and certain approximations (both theoretical and calculated), we proceed to compute the geoid height, referred to as the approximate geoid height. This process is detailed in [Kiamehr, 2006).

$$\tilde{N} = c\sum_{n=2}^{\infty}\left(\frac{2}{n-1} - Q_n^L - S_n^*\right)(\Delta g_n + \varepsilon_n^T) + c\sum_{n=2}^{M}(Q_n^L + S_n^*)(\Delta g_n + \varepsilon_n^S) \quad (4)$$

where $\varepsilon_n^T, \varepsilon_n^S$ – are spectral errors of ground and global gravity anomalies, respectively.

Since the true values of the error components are unknown, their estimation can be based on some standard approaches and stochastic models.

The key factor to minimise the global RMS and reduce all relevant errors in geoid modelling is a suitable choice of LS parameters.

The geoid model was computed for a test area bounded by coordinates:

$40 \le \varphi \le 46$ latitude and $66 \le \varphi \le 71$ longitude. Calculation of height anomalies was performed according to the block diagram of the calculation algorithm shown in Figure 1. in the LSMSSOFT programme. (Ågren, 2004).

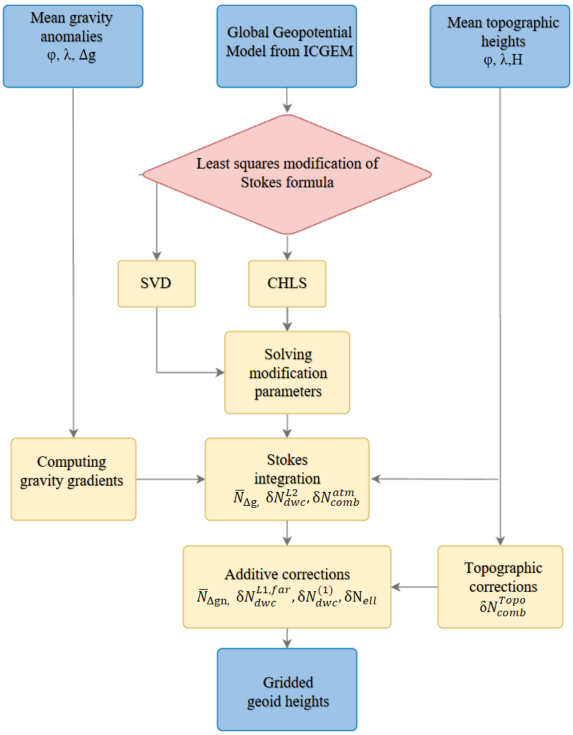


Figure 1. Block diagram of the calculation algorithm in LSMSSOFT software (Ågren, 2004)

Geoid models for the test site were calculated using different maximum degrees of spherical harmonics of global gravity model (GGM) coefficients and spherical integration radii using LSMSSOFT software. Geoid model calculations included the use of XGM2019 spherical harmonic coefficients up to degrees 120, 200, 300, 400, 500, 630 with a combination of ground gravity data error variance C(0) - 0.5, 1, 3, 6, 9, 16 mGal^2 . The results are summarised in Table 1 and Figure 2.

Table 1 Statistics of fitting the combination of spherical harmonic coefficients with error variance of ground gravimetric data

	C(0), mGal^2 .									
	16		9		6		3		1	
Mmax	STD	RMSE	STD	RMSE	STD	RMSE	STD	RMSE	STD	RMSE
630	0,259	0,284	0,220	0,244	0,216	0,236	0,212	0,225	0,209	0,214
500			0,216	0,229	0,213	0,223	0,210	0,218	0,208	0,211
400									0,208	0,210
360									0,207	0,208
300									0,206	0,205
200									0,203	0,272
180									0,202	0,274

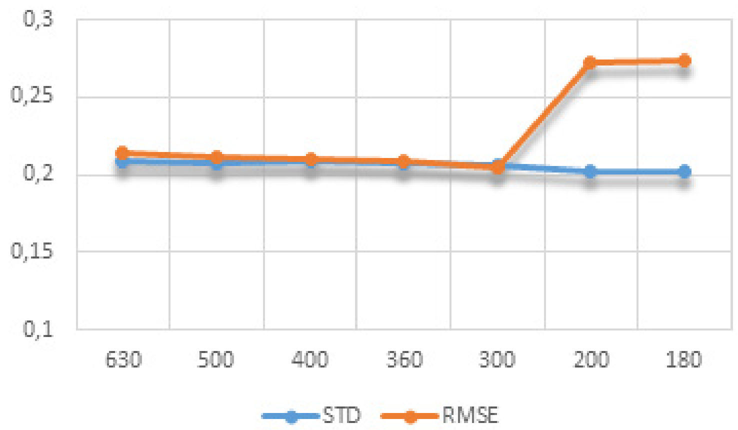


Figure 2 - Graph of statistic change with decreasing degree of GGM with error variance of ground gravimetric data C(0)=1 mGal^2

Based on the obtained results, value matrices were utilized as inputs for the final version of the model:

– Combination of terrestrial gravimetric - Digitized gravity maps and WGM2012 data;

– Harmonics of the global gravity model - XGM2019;

– Digital elevation model - GLO30.

The initial parameters for calculating the modification parameters were:

– Degree of modification L=M=300;

– Error variance of ground gravimetric data C(0)=1 mGal^2;

– Integration cap size ψ=1°.

The preliminary model of the geoid is given in Figure 3, Geoid heights for a given area vary in the range from -44 till -33 meters.

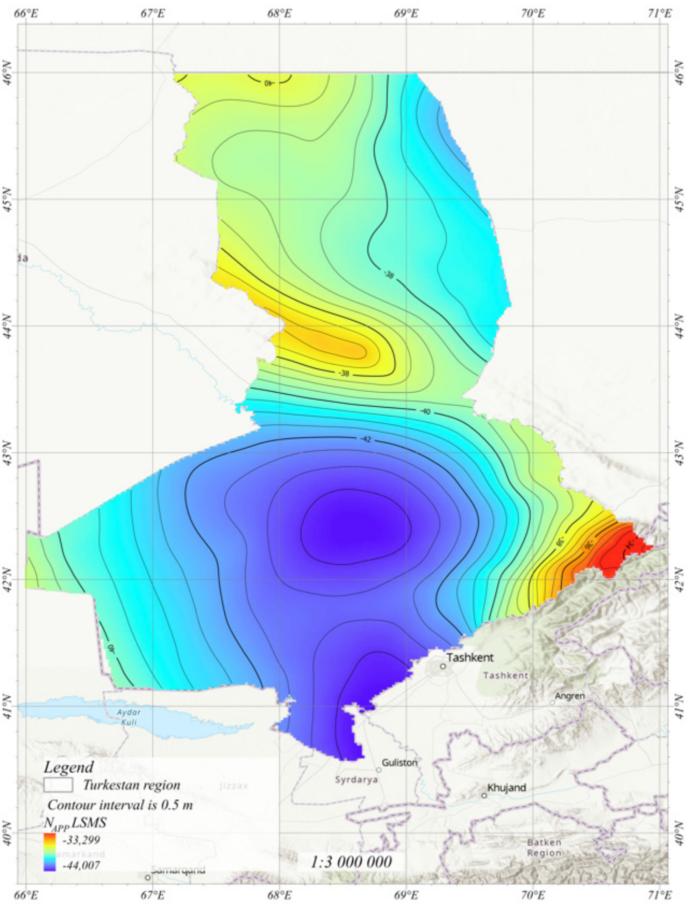


Figure 3 - Approximate geoid heights without Napp corrections (first approximation model)

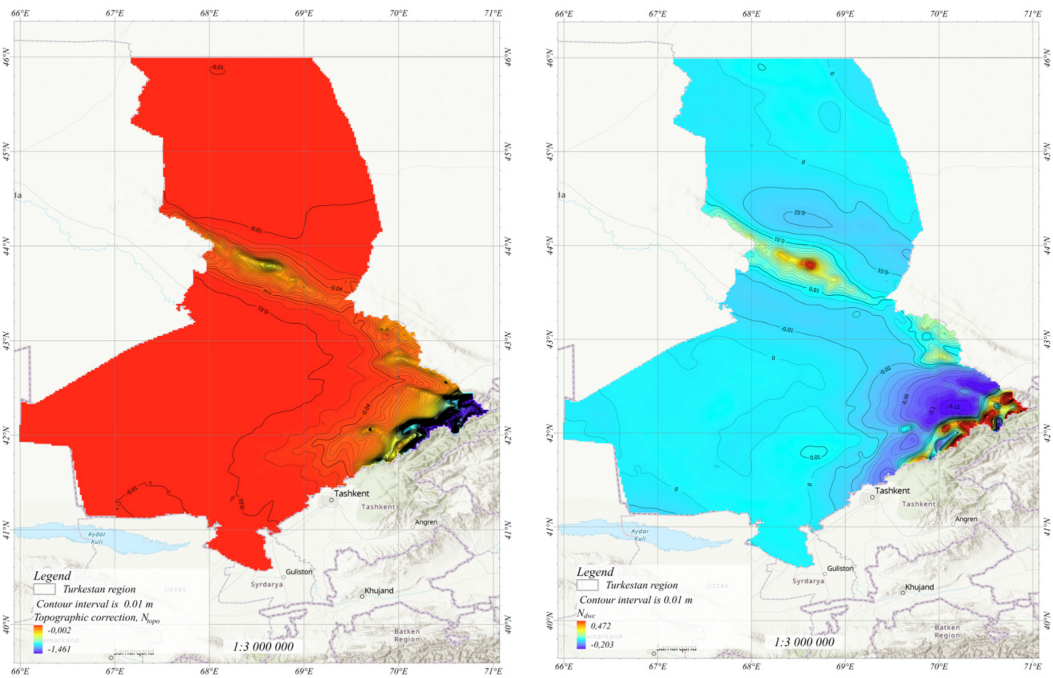
Additive corrections. When using Stokes' formula to define the geoid, it is crucial to ensure that there are no external masses beyond the geoid and that gravity data is reduced to sea level. However, the existence of topographic and atmospheric masses above the geoid requires the addition of specific corrections to satisfy these conditions.

The computational procedure for estimating the final geoid height can be performed using the following formula:

(5)

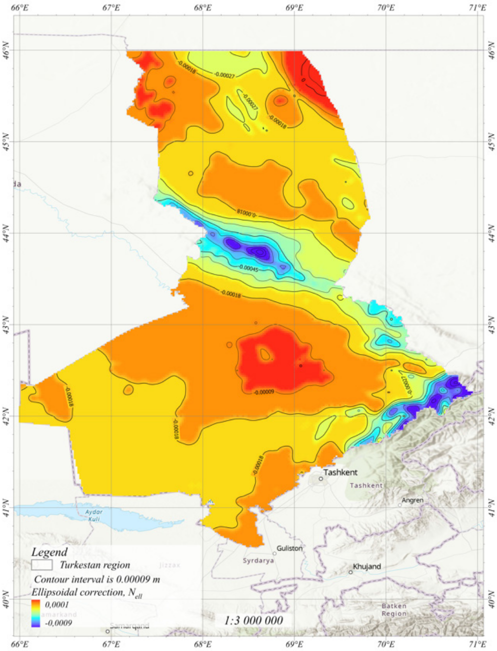
where δ- is a combined topographic correction that includes the sum of the direct and indirect topographic effects on geoid height, δ - downward continuation effect, - is a combined atmospheric correction including the sum of direct and indirect atmospheric effects, and δ - ellipsoidal correction for the spherical approximation of the geoid in the Stokes formula to an ellipsoidal reference surface.

Additive corrections for the geoid model were also calculated using LSMSSOFT. The calculation results are shown in Figure 4.

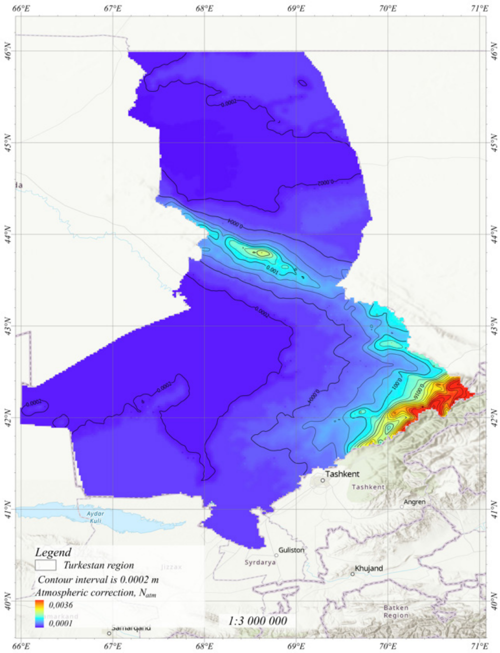


a) Combined topographic correction

b) Correction for analytical continuation downwards



c) Ellipsoidal correction



d) Combined atmospheric correction

Figure 4 - Corrections to approximate geoid heights using the

a) topographic correction, b) correction for analytical downward continuation, c) ellipsoidal correction, d) atmospheric correction.

The topographic correction, illustrated in Figure 4a, ranges from -1.9671 m to -0.0004 m, with a mean value of -0.059 m and a standard deviation of 0.175 m. The minimum value of the DWC reduction is -0.458 m, the maximum is 1.102 m, the mean is 0.003 m, and the standard deviation is 0.068 m, as shown in Figure 4b. The ellipsoidal correction, depicted in Figure 4c, varies from -1.2 mm to 0.2 mm, with a mean of 0.19 mm and a standard deviation of 0.145 mm. The atmospheric correction across the study area is minimal, with values ranging from 0.1 mm to 4.2 mm, a mean of 0.48 mm, and a standard deviation of 0.55 mm, as shown in Figure 4d. The overall correction values for the test area have a minimum of -1.822 m, a maximum of 0.031 m, a mean of -0.062 m, and a standard deviation of 0.15 m.

Figure 5 illustrates the outcomes of the calculations performed using the Least Squares Modification of Stokes formula with additive corrections, the geoid model for the test area that covers an expanse exceeding 110,000 km².

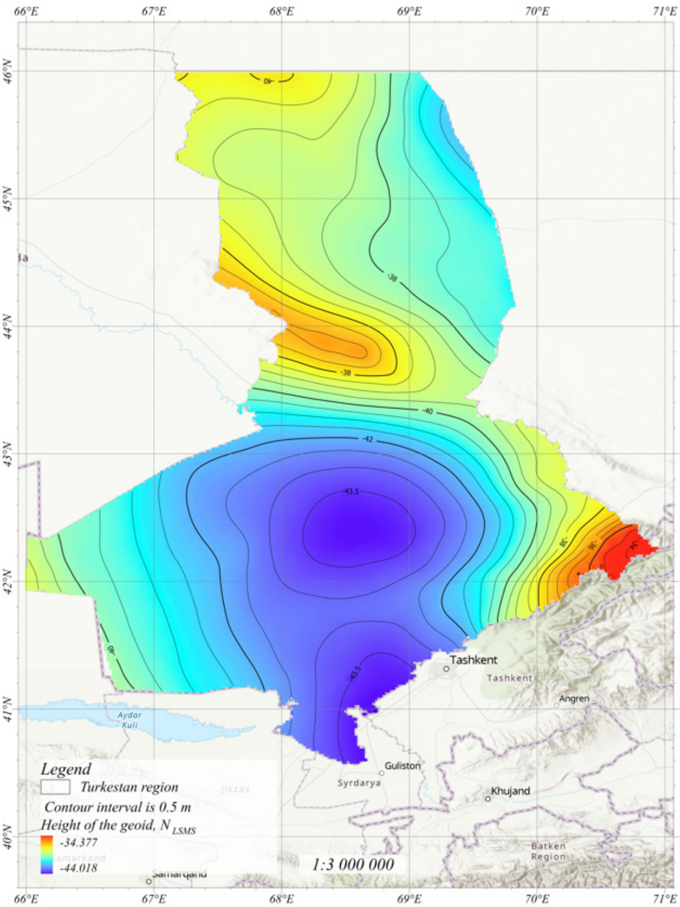


Figure 5 - Height of geoid according to the method of Modification of Stokes formula by the method of least squares, N_{LSMS}

The calculated geoid heights at the test site using the LSMS method range from -48.244 m to -32.776 m, with a mean of -40.405 m and a standard deviation of 2.210 m.

Results

The accuracy of the geoid model was assessed by interpolating heights to GNSS/levelling control points. Statistics of deviations N_{geoid} -$N_{GNSS/Lev}$, which are indicators of model errors at control points, are presented in Tables 2, 3.

Table 2. Statistics of differences between geoid heights $N_{GNSS/lev}$ and $N_{Geoid,}$ obtained from different calculation methods (unit, m)

Methods	Min	Max	Mean	STD	RMSE
N_{LSMS}	-0,8134	0,3799	0,0002	0,2061	0,2052
N_{CSH}	-0,8269	0,3992	-0,0156	0,2039	0,2035

Table 3: Range of values of geoid height differences $N_{GNSS/lev}$ and $N_{Geoid,}$ (unit, %)

GGM	⩽ -0.4м	⩽ -0,3м	⩽ -0,2м	⩽ -0,1м	⩽ **0м**	⩽ 0,1м	⩽ 0,2м	⩽ 0,3м	⩽ 0,4м
N_{LSMS}	7,48	0,93	1,87	6,54	15,89	43,93	15,89	4,67	2,80
N_{CSH}	7,48	0,93	1,87	5,61	20,56	43,93	14,02	3,74	1,87

Table 3 shows that more than 75% of the points accuracy lies between -0.1 and 0.2, which shows good convergence.

The accuracy of a gravimetric geoid model is typically evaluated using statistical measures such as standard deviation (STD) and root mean square error (RMSE). These metrics quantify the discrepancies between the gravimetric geoid model and the geometric geoid model, providing a rigorous assessment of the model's precision (Pa'suya et al., 2022). By estimating the RMS values of the differences and considering the error ranges, the best solution for the gravimetric geoid model can be found. However, direct comparison of geoid models may not be feasible due to the presence of possible systematic errors (i.e., biases). The offset and slope of the raw data in the coordinate directions of the global system axis, representing the zero- and first-degree harmonics, are missing parameters of the gravimetric geoid. Various corrective surface models can be used to account for these systematic errors.

Table 4 - Statistics after application of the correction surface for geoid heights using the method N_{LSMS}

Method	Statistics	Before correction	After correction		
			4 parameters	5 parameters	7 parameters
N_{LSMS}	Min	-0.813	-0.732	-0.737	-0.694
	Max	0.380	0.413	0.419	0.450
	Mean	0.0002	-9.47e-07	2.88e-11	-3.59e-09
	STD	0.206	0.201	0.199	0.195

The standard deviation of the adjusted residual values is conventionally regarded as an external metric for assessing the absolute accuracy of the geoid model. Table 4 presents a detailed statistical analysis of the results obtained by applying correction surfaces to the computed geoid model.

Conclusion

For the initial calculation of the local geoid for the test area, the Least Squares Modification of Stokes' formula with Additive corrections (LSMSA), also known as the KTH-method developed by professor L Sjoberg, was employed. To mitigate systematic errors between gravimetric geoid heights and those derived from GNSS/levelling, four-parameter, five-parameter, and seven-parameter models were utilized. Among these, the seven-parameter model demonstrated the highest accuracy, achieving a reduction in the standard deviation by approximately 1 cm. The accuracy evaluation results showed that the method has an accuracy of 0.195 m, indicating that careful selection of data and internal calculation parameters is required. In the LSMSA method, the accuracy of the results obtained was affected by the selection of modification parameters. It was found that the earth gravity data error was the main parameter affecting the accuracy of the LSMSA method. This indicates that it is necessary to evaluate the accuracy of gravity data. Improving the density and accuracy of terrestrial gravity data can be an effective way to improve the accuracy of the geoid model using this method.

The development of local geoid model will allow to further scale the results of research work throughout the country and introduce the methodology of building a geoid model in public and private organisations engaged in the development and use of spatial data. Since officially there are no working analogues on the territory of Kazakhstan, the developed model is relevant and competitive within the country.

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