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IMMISCIBLE DISPLACEMENT OF HERSCHEL-BULKLEY FLUIDS IN POROUS MEDIA: A MODIFIED BUCKLEY-LEVERETT APPROACH

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Abstract. Non-Newtonian fluids are very common in oil field development and enhanced oil recovery (EOR) methods. The Herschel-Bulkley model is one of the most general and complex models that describe the flow of non-Newtonian fluids. While single-phase and multiphase flows of simple types of non-Newtonian fluids like the Bingham plastics and Power-law fluids in porous media has been deeply studied in the previous literature, the immiscible displacement of Herschel-Bulkley fluids by other Herschel-Bulkley fluids has remained underexplored. This study is an extension of the Buckley-Leverett theory that analytically describes the mentioned displacement processes assuming steady-state, incompressible flow in a linear homogeneous reservoir.

The proposed mathematical model uses a modified Darcy law for non-Newtonian fluids and Corey correlations to construct the fractional flow curve and its derivative by saturation. The impact of various non-Newtonian parameters, such as power-law index, consistency index and the initial pressure gradient of both the displaced and the displacing fluids on the fractional flow curve and its derivative is investigated. Then such reservoir development efficiency indicators as total oil production, sweep efficiency, displacement efficiency and oil recovery factor are calculated.

It is concluded that the increase of power-law index, consistency index and initial pressure gradient of the displacing fluid, and the decrease of those parameters for the displaced fluid leads to maximum development efficiency. The calculations have also shown that most of the extractable resources are produced before breakthrough, and the time required to produce all extractable resources is significantly higher than the breakthrough time.

Keywords: Non-Newtonian fluids, Herschel-Bulkley model, Buckley-Leverett theory, Flow in porous media, Fractional flow theory, Immiscible displacement

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ГЕРШЕЛЬ-БАКЛИ СҰЙЫҚТЫҚТАРЫН БІРІКТІРМЕЙТІН ЫҒЫСТЫРУ ПОРЛЫ ОРТАСЫНДА: БАКЛИ-ЛЕВЕРЕТТТІҢ МОДИФИКАЦИЯЛАНҒАН МОДЕЛІ

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Аннотация. Ньютондық емес сұйықтықтар мұнай-газ кен орындарын игеру барысында және мұнай өндіруді арттыру (EOR) әдістерінде кеңінен қолданылады. Гершель–Балкли моделі Ньютондық емес сұйықтықтардың ағысын сипаттайтын ең жалпы әрі күрделі модельдердің бірі болып табылады. Кеуекті ортада Бингам пластиктері мен дәрежелік заңға бағынатын (power-law) сұйықтықтар сияқты Ньютондық емес сұйықтықтардың бірфазалы және көпфазалы ағындары бұрынғы зерттеулерде жан-жақты қарастырылғанымен, Гершель–Балкли сұйықтықтарының басқа Гершель–Балкли сұйықтығымен араласпайтын ығыстырылу үдерістері жеткілікті деңгейде зерттелмеген. Аталған зерттеу Бакли–Леверетт теориясының дамуы болып табылады және сызықтық біртекті қабатта орнықкан (стационарлық) сығылмайтын

ағын жағдайында көрсетілген ығыстыру үдерістерін аналитикалық түрде сипаттауға бағытталған.

Ұсынылған математикалық модель Ньютондық емес сұйықтықтарға арналған Дарси заңының модификацияланған түрін, үлестік ағын (fractional flow) қисығын және оның қанығу бойынша туындысын құрастыру үшін Кори корреляцияларын қолдануға негізделген. Сонымен қатар ығыстырылатын және ығыстырушы сұйықтықтардың әртүрлі Ньютондық емес параметрлерінің, атап айтқанда дәрежелік индексінің, консистенция индексінің және бастапқы қысым градиентінің үлестік ағын қисығына және оның туындысына әсері зерттеледі.

Одан әрі қабатты игеру тиімділігінің негізгі көрсеткіштері ретінде жалпы мұнай өндіру көлемі, қамту тиімділігі, ығыстыру тиімділігі және мұнай беру коэффициенті есептеледі. Зерттеу нәтижелері ығыстырушы сұйықтықтың дәрежелік индексі, консистенция индексі және бастапқы қысым градиенті артқан жағдайда, ал бұл параметрлер ығыстырылатын сұйықтық үшін кемігенде, игеру тиімділігінің ең жоғары мәндеріне қол жеткізілетінін көрсетті. Сонымен қатар есептеу нәтижелері өндірілетін ресурстардың басым бөлігі серпіліске (breakthrough) дейін алынатынын және барлық өндіруге жарамды ресурстарды толық игеруге кететін уақыт серпіліс уақытымен салыстырғанда едәуір ұзақ болатынын анықтады.

Түйін сөздер: Ньютондық емес сұйықтықтар, Гершель–Балкли моделі, Бакли–Леверетт теориясы, кеуекті ортадағы ағын, үлестік ағын теориясы, араласпайтын ығыстыру

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НЕСМЕШИВАЮЩЕЕСЯ ВЫТЕСНЕНИЕ ЖИДКОСТЕЙ ГЕРШЕЛЯ-БАЛКЛИ В ПОРИСТОЙ СРЕДЕ: МОДИФИЦИРОВАННАЯ МОДЕЛЬ БАКЛИ-ЛЕВЕРЕТТА

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Аннотация. Неньютоновские жидкости широко распространены при разработке нефтяных месторождений и применении различных методов повышения нефтеотдачи. Модель Гершеля-Балкли является одной из наиболее общих и комплексных моделей, описывающих течение неньютоновских жидкостей. В то время как однофазная и многофазная фильтрация простых типов неньютоновских жидкостей в пористой среде (например, бингамовских пластиков и жидкостей, подчиняющихся степенному закону) подробно изучена в литературе, несмешивающееся вытеснение жидкостей Гершеля-Балкли остаётся недостаточно исследованным. Настоящая работа представляет собой расширение теории Бакли-Леверетта, которая аналитически описывает процессы вытеснения при предположении установившейся фильтрации несжимаемой жидкости в линейном однородном коллекторе. Предложенная математическая модель основана на модифицированном законе Дарси для неньютоновских жидкостей и корреляциях Кори, используемых для построения кривой фракционного потока и её производной по водонасыщенности. Исследуется влияние неньютоновских параметров - степенного индекса, индекса консистенции и начального градиента давления - как для вытесняемой, так и для вытесняющей жидкости, на кривую фракционного потока и её производную. Далее рассчитываются показатели эффективности разработки пласта, включая суммарную добычу нефти, эффективность охвата, эффективность вытеснения и коэффициент извлечения нефти. Показано, что увеличение степенного индекса, индекса консистенции и начального градиента давления вытесняющей жидкости, а также снижение этих параметров для вытесняемой жидкости обеспечивают максимальную эффективность разработки. Расчёты также демонстрируют, что основная часть извлекаемых ресурсов добывается до прорыва, тогда как время, необходимое для извлечения всего объёма извлекаемых ресурсов, существенно превышает время прорыва.

Ключевые слова: неньютоновские жидкости, модель Гершеля-Балкли, теория Бакли-Леверетта, фильтрация в пористой среде, теория фракционного потока, несмешивающееся вытеснение

Introduction. Non-Newtonian fluids are very common in oil field development. Most of the oil reserves in the world today are considered unconventional reserves, which include non-Newtonian oils (Yatimi et al., 2024). Such oils are also known as heavy oils, and they usually do not flow immediately after the pressure gradient exceeds the value of zero. Instead, in order for the flow to occur for such oils the pressure gradient needs to exceed a certain value that is dependent on the property called yield stress. Such oils may be described by the Bingham model (Sun et al., 2024). Polymer solutions that are usually injected into an oil reservoir to enhance the oil recovery are also considered non-Newtonian and can be described by the Power-Law model (Yao and Ge, 2011). On the other hand, foams that are also used in EOR methods, some fluids used in hydraulic fracturing and some drilling muds can be described by the Herschel-Bulkley model (Balhoff et al., 2011). Other use cases of non-Newtonian fluids include injection of cement in soils and penetration of glue in porous substrates.

Immiscible flow of two or more fluids in porous media is quite common in subsurface systems. Compared to single phase flow, the multiple-phase flow is much complex and not fully understood. The fundamentals of understanding multiphase flow in porous media were established by Buckley and Leverett in their fractional flow theory (Muhammad and Moataz, 2022). Later, a graphical method of determination of average displacing fluid saturation based on the Buckley-Leverett theory was proposed by Welge (Civan, 2011). While most of the studies on the flow of non-Newtonian fluids in porous media have focused on single phase flow, some authors have also studied multiphase flow of non-Newtonian fluids.

In the work (Wu et al., 1991), an immiscible displacement of a Newtonian fluid by a non-Newtonian power-law fluid has been studied. The practical use case of this study is injection of polymer solutions (which are usually power-law fluids) into a reservoir that contains Newtonian oil. The authors have proposed an analytical extension of the Buckley-Leverett theory for power-law fluids by using a modified Darcy law and have analyzed this flow in a linear reservoir. In the work (Pruess and Wu, 1996), multiphase flow of non-Newtonian power-law and Bingham fluids in porous media has been studied using a numerical simulation method in 3D. A five-spot system with 4 injection wells and 1 production well has been considered.

In the work (Ter Haar, 2018), an injection of non-Newtonian power-law foams using the surfactant-alternating-gas (SAG) method has been considered. The authors also extended the Buckley-Leverett theory; however, they have analyzed injection in a circular reservoir instead of a linear one. In (Huang et al., 2017), an analytical solution describing a non-Darcy displacement of Newtonian fluid by a non-Newtonian power-law fluid in a linear reservoir is presented. The non-Darcy flow is described by the Barree-Conway model (Barree and Conway, 2009) and it is useful when describing the flow of fluids in porous media at very high flow rates when deviations from the Darcy law can be observed (Bear, 2013).

The aim of this work is to extend the Buckley-Leverett theory in order to analytically describe the immiscible displacement of one non-Newtonian Herschel-Bulkley fluid by another Herschel-Bulkley fluid in a linear reservoir, which, to our best knowledge has not been considered yet in previous literature. Our goal is to study the effect of various Herschel-Bulkley model parameters (for both displacing and displaced fluids) on the efficiency of oil extraction. The proposed analytical model offers a flexible framework for analyzing oil displacement processes, as it can be easily adapted to represent Newtonian, Bingham, or Power-Law fluids by adjusting the Herschel-Bulkley model parameters.

Materials and methods. In a cross section of a linear reservoir where immiscible displacement of 1 fluid by another takes place, both fluids are present (meaning that multiphase flow occurs), but they do not mix and each of them has their own saturation which affects the velocity at which they flow. Therefore, the Darcy velocity of each fluid has to be modified in such a way that it takes into account multiphase flow. This is done by including relative permeability of each fluid in the Darcy law, so that for the displaced fluid it becomes (Wu, 2016):

$$v_o = -\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}, \tag{1}$$

where k is the rock absolute permeability, μ_o is the viscosity of the displaced fluid, k_{ro} is the relative permeability of the displaced fluid, and $\frac{\partial P_o}{\partial x}$ is the pressure gradient of the displaced fluid. Similarly, for the displacing fluid we get:

$$v_w = -\frac{kk_{rw}}{\mu_w}\frac{\partial P_w}{\partial x}. \tag{2}$$

In a linear reservoir, the flow rates of each phase become:

$$Q_o = -A\frac{kk_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}, \tag{3}$$

$$Q_w = -A\frac{kk_{rw}}{\mu_w}\frac{\partial P_w}{\partial x}, \tag{4}$$

where A stands for the cross-sectional area of the linear reservoir. The total flow rate through the cross-sectional area of the reservoir can be expressed as:

$$Q_t = Q_o + Q_w. \tag{5}$$

Taking into account the aforementioned equations, we can do the following transformation:

$$Q_w\left(\frac{\mu_w}{kk_{rw}} + \frac{\mu_o}{kk_{ro}}\right) = \frac{Q_t\mu_o}{kk_{ro}} + A\frac{\partial P_c}{\partial x}, \tag{6}$$

where $\frac{\partial P_c}{\partial x}$ is the capillary pressure gradient and is defined as:

$$\frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x}. \tag{7}$$

The capillary pressure stands for the pressure difference at the border of 2 phases which are separated by a curved surface. The capillary pressure is proportional to the surface tension between 2 phases, and reverse-proportional to the radius of the capillary (Fanchi, 2006). In general, the average capillary pressure in the reservoir depends on the saturation of the wetting phase in each part of the reservoir. If this saturation is small, then the wetting phase occupies firstly the smallest pores (capillaries of the smallest radius) and that is why the average capillary pressure becomes higher. When the saturation of the wetting phase increases then it also starts occupying bigger parts, and as a result, the average capillary pressure decreases. Therefore, there is a reverse-proportional relationship between the wetting phase saturation and the capillary pressure (Ahmed, 2019).

In this work we assume that the displacing fluid is the wetting phase. A typical profile of displacing fluid saturation during immiscible displacement looks as shown on Figure 1. A typical relationship between the wetting phase saturation and the capillary pressure in a reservoir is shown on Figure 2 (Ahmed, 2019). Here S_{wc} stands for the residual (not displaceable) saturation of the reservoir with the wetting phase. S_{or} stands for the residual saturation of the non-wetting phase. Therefore, the term 1 - S_{or} stands for the maximum achievable saturation of the displacing fluid. S_{wf} stands for the saturation of the displacing fluid at the displacement front.

The capillary pressure gradient can be expressed as follows:

$$\frac{\partial P_c}{\partial x} = \frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}. \quad (8)$$

Therefore, by knowing the relationship between the capillary pressure and wetting phase saturation and the distribution of the wetting phase saturation in the reservoir, we can find the capillary pressure gradient. The relationship between the capillary pressure and the wetting phase saturation is often displayed with the help of a function called Leverett J-function (Armstrong et al., 2019):

$$J(S_w) = \frac{P_c(S_w)}{\sigma \cos\theta}\sqrt{\frac{k}{\phi}}, \quad (9)$$

where ϕ is rock porosity, σ is the surface tension, θ is the contact angle, $J(S_w)$ is the J-function that is determined empirically for a rock sample taken from the reservoir. Based on the experiments, a relationship between S_w and $J(S_w)$ is obtained and after that the relationship between the capillary pressure and the wetting phase saturation is obtained.

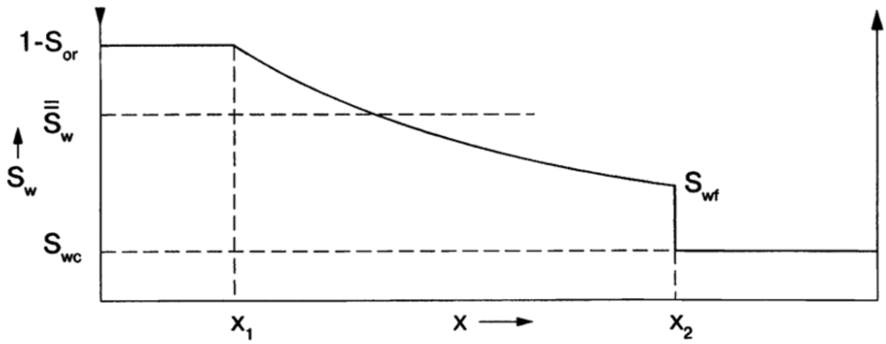


Figure 1 – Typical saturation profile of displacing fluid in reservoir.

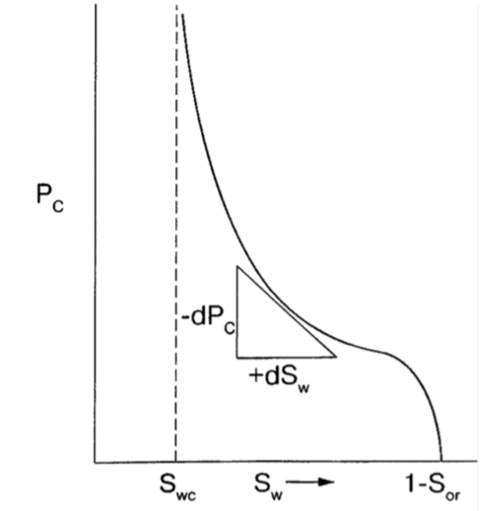


Figure 2 – Typical relationship between capillary pressure and saturation of displacing fluid.

Let's denote the fractional flow of displacing fluid by the symbol f_w at any point in the reservoir. It is defined as:

$$f_w = \frac{Q_w}{Q_w + Q_o} = \frac{Q_w}{Q_t}. \tag{10}$$

By using equation (6) we get:

$$f_w = \frac{1 + \frac{k k_{ro} A}{\mu_o Q_t} \frac{\partial P_c}{\partial x}}{1 + \frac{\mu_w k_{ro}}{\mu_o k_{rw}}}. \tag{11}$$

The distribution of displacing fluid saturation in the reservoir, and therefore the term $\frac{\partial s_w}{\partial x}$ can be determined from the Buckley-Leverett theory (Muhammad and Moataz, 2022), where it is assumed that each displacing fluid saturation value has its own velocity of moving towards the other end of the linear reservoir and saturation is considered to be reversely proportional to this velocity. Mathematically, this is expressed as follows (Ahmed, 2019):

$$v_{S_w} = \frac{dx}{dt}(S_w) = \frac{Q_t}{A\phi} \frac{df_w}{dS_w}(S_w). \tag{12}$$

Integration of equation (12) leads to the following:

$$x(S_w) = \frac{1}{A\phi} \frac{df_w}{dS_w}(S_w) \int_0^t Q_t dt = \frac{W_i}{A\phi} \frac{df_w}{dS_w}(S_w), \tag{13}$$

where W_i is the total injected volume of displacing fluid at a certain time moment. The physical meaning of equation (12) is that the velocity $\frac{dx}{dt}(S_w)$ of a certain saturation increases with the increase of derivative of fractional flow by saturation for this saturation. Equation (13) shows the current position of a certain displacing fluid saturation in a linear reservoir. The relationship between the fractional flow derivative by saturation from saturation usually looks like an example shown on Figure 3 (a). On the other hand, Figure 3 (b) shows the relationship between the coordinate x and the saturation, which is built based on the equation (13) (Ahmed, 2019).

It is important to note that according to the Buckley-Leverett theory, the displacing fluid saturation on the displacement front cannot be less than the saturation that corresponds to the inflection point of the curve $\frac{df_w}{dS_w}(S_w)$. This is related to the fact that if such values of saturation were allowed, then 2 different values of saturation would have corresponded to the same coordinate x, which is physically not possible. That is why, in Buckley-Leverett theory, the displacement front saturation is determined by marking 2 equal areas A and B, as Figure 3 (b) shows.

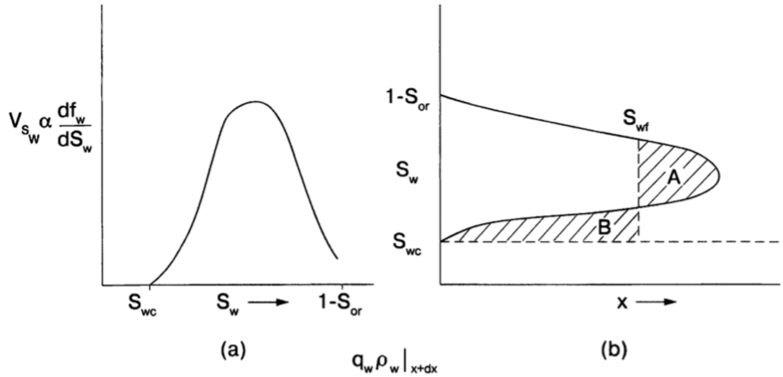


Figure 3 – Typical relationship between fractional flow derivative and saturation of displacing fluid (a); Determination of saturation at the displacement front according to the Buckley-Leverett theory (b)

In order to construct a relationship between the derivative of fractional flow by displacing fluid saturation and displacing fluid saturation, we first need to find the relationship between fractional flow and displacing fluid saturation, which can be obtained from equation (11). For this we need to express the relative permeabilities k_{ro} and k_{rw} through the displacing fluid saturation. The increase of saturation of the displacing fluid increases its relative permeability and decreases the relative permeability of the displaced fluid. There are many empirical relationships that relate the displacing fluid saturation to the relative permeabilities of phases in multiphase flow. The much more problematic part is expressing the term $\frac{\partial P_c}{\partial x}$ in equation (11) through saturation. As shown before, the term $\frac{\partial P_c}{\partial x}$ can be expressed as $\frac{dP_c}{dS_w}\frac{\partial S_w}{\partial x}$. Then we could have found the term $\frac{dP_c}{dS_w}$ from equation (9) by knowing the relationship between the saturation and the J-function. However, the problematic part is the term $\frac{\partial S_w}{\partial x}$. The problem is that in order to determine the distribution of displacing fluid saturation in the reservoir we need to know the value of $\frac{df_w}{dS_w}$ which we are trying to find. This strongly complicates finding an analytical solution for this problem. Due to this fact, from this point, in this work we will assume that the capillary pressure gradient is very small so that it can be ignored. Generally speaking, it is a quite justified assumption, especially in points with high wetting phase saturation. Several authors in previous literature have also assumed capillary pressure to be negligibly small (Ahmed, 2019; Luis et al., 2022). In other words, we will assume that:

$$\frac{\partial P_o}{\partial x} = \frac{\partial P_w}{\partial x} = \frac{\partial P}{\partial x}, \tag{14}$$

As a result, for the case when both the displacing and displaced fluids are Newtonian we would get the following equation for the fractional flow:

$$f_w = \frac{1}{1 + \frac{\mu_w k_{ro}}{\mu_o k_{rw}}}. \tag{15}$$

Using equation (15) and the empirical relationship between the displacing fluid saturation and the relative permeabilities, we can construct a relationship between fraction flow and the displacing fluid saturation. This would allow to find the relationship between the derivative of fractional flow and the displacing fluid saturation, and therefore, to use the Buckley-Leverett theory to find important reservoir development parameters, such as current position of the displacement front, total oil production and etc.

Now let's determine the modified Darcy law for multiphase flow of Herschel-Bulkley fluids. As mentioned before, we assume that both the displacing and the displaced fluid are non-Newtonian fluids that can be described by the Herschel-Bulkley model. A practical use case for this could be the displacement of a non-Newtonian oil by a polymer solution. For single phase flow on Herschel-Bulkley fluid in porous media, the following modified Darcy law is most commonly used in literature, which also aligns well with the experiments (Sánchez-Vargas et al., 2023):

$$v = \left(\frac{k}{\mu_{ef}}\left(-\frac{\partial P}{\partial x} - G_0\right)\right)^{\frac{1}{n}}, \tag{16}$$

where G_0 is the initial pressure gradient, n is the power-law index, μ_{ef} is the effective viscosity. The initial pressure gradient stands for the minimal pressure gradient that needs to occur in the fluid in order for it to start moving. The power-law index is a coefficient that takes into account non-linear relationship between shear stress and shear rate for some non-Newtonian fluids. For the flow to occur the pressure gradient $\frac{\partial P}{\partial x}$ needs to exceed the value of G_0. The initial pressure gradient G_0 can be determined experimentally, however, several empirical correlations and analytical formulas have been proposed in literature for its determination. For instance, experiments conducted in the work (Gafarov and Shamaev, 2005) have shown that for sands the value of G_0 can be determined as (SI units used – G_0 is measured in Pa/m):

$$G_0 = \frac{0.052\tau_0}{k^{0.62}}, \tag{17}$$

and for carbonate rocks as:

$$G_0 = \frac{0.003\tau_0}{k^{1.596}}. \tag{18}$$

Here τ_0 represents the yield stress of the non-Newtonian fluid. The effective viscosity is determined from the formula:

$$\mu_{ef} = \frac{K}{12}\left(9 + \frac{3}{n}\right)^n (150k\phi)^{\frac{1-n}{2}}, \tag{19}$$

where K stands for the consistency index, which is 1 of the 3 key parameters of the Herschel-Bulkley model and ϕ is the porosity.

Now we need to modify the equation (16) to account for the multiphase flow and express it in terms of the pressure gradient. As a result, for the displaced fluid we get:

$$-\frac{dP}{dx} = G_{0_o} + \frac{{v_o}^{n_o}\mu_{ef_o}}{k k_{ro}}. \tag{20}$$

For a linear reservoir the Darcy velocity of the displaced fluid can be expressed as follows:

$${v_o}^{n_o} = \frac{{Q_o}^{n_o}}{A^{n_o}}. \tag{21}$$

By inserting equation (21) into equation (20) we get:

$$-dP = G_{0_o}\,dx + \frac{{Q_o}^{n_o}\mu_{ef_o}}{A^{n_o} k k_{ro}}\,dx. \tag{22}$$

In this case we assume that:

$$-\frac{dP}{dx}=\frac{\Delta P}{L}. \qquad (23)$$

This linear approximation of the pressure gradient is valid since steady-state is assumed and the reservoir is considered linear. ΔP corresponds to the difference between pressures at both ends of the linear reservoir and L is the length of the reservoir. Then we get:

$$\Delta P = G_{0_o}L+\frac{{Q_o}^{n_o}\mu_{ef_o}}{A^{n_o}kk_{ro}}L. \qquad (24)$$

By solving for the flow rate of the displaced fluid we get:

$$Q_o = A\left(\frac{(\Delta P - G_{0_o}L)kk_{ro}}{\mu_{ef_o}L}\right)^{\frac{1}{n_o}}. \qquad (25)$$

Similarly for the displacing fluid we get:

$$Q_w = A\left(\frac{(\Delta P - G_{0_w}L)kk_{rw}}{\mu_{ef_w}L}\right)^{\frac{1}{n_w}}. \qquad (26)$$

Then the fractional flow equation would look as follows:

$$f_w=\frac{Q_w}{Q_w+Q_o}=\frac{\left(\frac{(\Delta P - G_{0_w}L)kk_{rw}}{\mu_{ef_w}L}\right)^{\frac{1}{n_w}}}{\left(\frac{(\Delta P - G_{0_w}L)kk_{rw}}{\mu_{ef_w}L}\right)^{\frac{1}{n_w}}+\left(\frac{(\Delta P - G_{0_o}L)kk_{ro}}{\mu_{ef_o}L}\right)^{\frac{1}{n_o}}}. \qquad (27)$$

In case if $G_{0_w}=0, G_{0_o}=0, n_w=1, n_o=1$ then equation (27) becomes the same as equation (15). The analysis of equation (27) shows that the only variables that change in the direction of flow are the relative permeabilities k_{ro} and k_{rw}. In this case the effective viscosities should also be modified to account for the multiphase flow as follows:

$$\mu_{ef_o}=\frac{K_o}{12}\left(9+\frac{3}{n_o}\right)^{n_o}(150kk_{ro}\phi)^{\frac{1-n_o}{2}}, \qquad (28)$$

$$\mu_{ef_w}=\frac{K_w}{12}\left(9+\frac{3}{n_w}\right)^{n_w}(150kk_{rw}\phi)^{\frac{1-n_w}{2}}. \qquad (29)$$

The relative permeabilities can be expressed as functions of only displacing fluid saturation, which on the other hand, is a function of coordinate x and time.

Several empirical correlations exist that show the relationship between relative permeabilities during multiphase flow and displacing fluid saturation. It is important to note that conducting in experiment in a laboratory for a specific rock sample taken from the reservoir in order to determine the relative permeability curves is always more accurate than using an empirical correlation, however, such conditions are not always available. In this work we are going to use the most commonly used empirical correlation known as the Corey correlation (Goda and Behrenbruch, 2004). According to this correlation, the relative permeability of the displaced fluid is determined as follows:

$$k_{ro} = \left(1 - \frac{S_w - S_{wc}}{1 - S_{wc}}\right)^2 \left(1 - \left(\frac{S_w - S_{wc}}{1 - S_{wc}}\right)^2\right). \qquad (30)$$

The relative permeability of the displacing fluid is determined as:

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wc}}\right)^4. \qquad (31)$$

In this work we assume that $S_{wc} = 0.1$. In that case, the relative permeability curves, constructed based on equations (30) and (31) would look like Figure 4 shows:

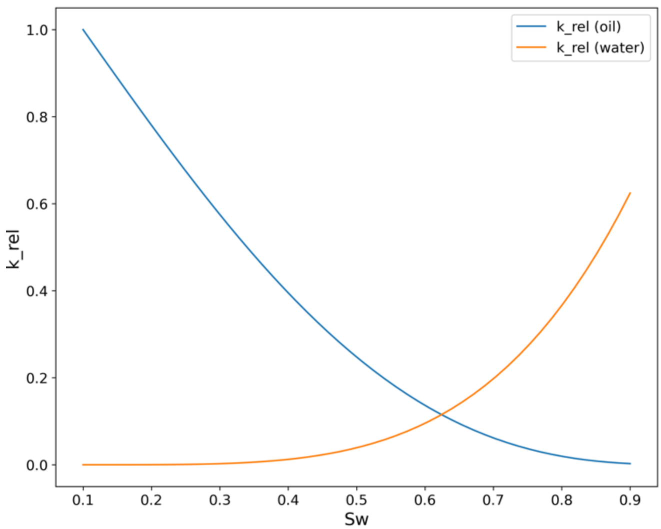


Figure 4 – Relative permeability curves according to the Corey correlation

Results and discussion. The next goal is to construct a sample relationship between the fractional flow and the displacing fluid saturation using the relative permeability curves in order to validate the proposed mathematical model. For this we will use the parameter values that are shown in Table 1.

Table 1 - Parameter values used to construct the relationship between the fractional flow and saturation

Parameter name	Parameter value
$\Delta P, Pa$	10000000
L, m	100
k, m^2	$0.1 * 10^{-12}$
A, m^2	400
ϕ	0.1
$G_{0_w}, Pa/m$	3000
$K_w, Pa * s^{0.95}$	$2 * 10^{-3}$
n_w	0.95
$G_{0_o}, Pa/m$	10000
$K_o, Pa * s^{1.05}$	$5 * 10^{-3}$
n_o	1.05
S_{wc}	0.1

Figure 5 shows the relationship between fractional flow of displacing fluid and saturation of displacing fluid built based on equation (27) using the values of parameters from Table 1.

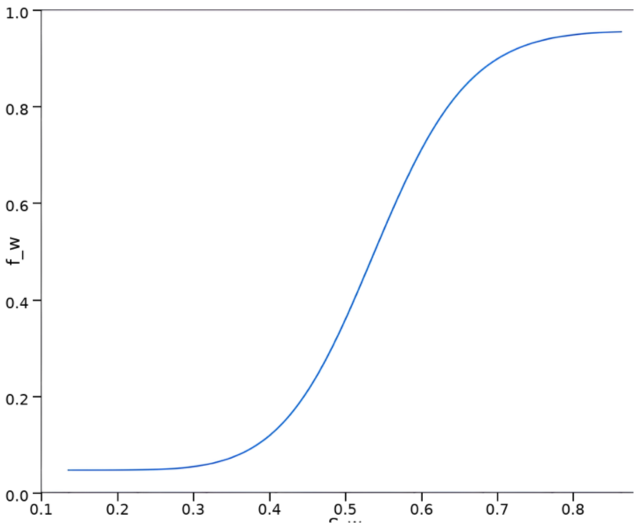


Figure 5 – Fractional flow curve built based on parameters from Table 1

Now let's investigate the impact of various parameters on the fractional flow curve. Figure 6 (a) shows the relationship between fractional flow and saturation for various values of the power-law index of displacing fluid which change in the range 0.5-3. The graph shows that the decrease of power-law index of displacing fluid increases its fractional flow. The most abrupt increase of fractional flow can be noticed when the power-law index falls below 1.

Figure 6 (b) shows the relationship between fractional flow derivative and saturation for various values of power-law index of displacing fluid. As mentioned earlier in this work, the velocity of frontal advance for a certain saturation increases with the increase of the value of fractional flow derivative for this saturation. Therefore, from Figure 6 (b) we can conclude that in general, the decrease of power-law index of displacing fluid increases the frontal advance velocity of saturation at the displacement front. An interesting observation is that for the values of power-law index of displacing fluid higher than $n_w = 3$, an inflection point can be noticed after which the value of derivative of fractional flow (and therefore, frontal advance velocity) increases with the increase of the displacing fluid power-law index.

Figure 7 (a) shows the relationship between the fractional flow and saturation for various values of power-law index of displaced fluid n_o. It can be seen that the decrease of n_o leads to the decrease of fractional flow of displacing fluid. The most abrupt decrease of fractional flow occurs when the value of n_o drops below 1. So, we can conclude that the increase of n_o and n_w above 1 does not that strongly impact the fractional flow, as the decrease below 1.

Figure 7 (b) shows the relationship between fractional flow derivative and saturation for various values of power-law index of displaced fluid. Just as for the case with n_w, the decrease of the value of n_o increases the value of fractional flow derivative. However, the key difference is that for n_w the maximum fractional flow derivative corresponds to much lower displacing fluid saturation at the front compared to n_o. This means that for lower power-law indexes of displaced fluid the displacing fluid saturation at the displacement front would be much higher than for lower power-law indexes of displacing fluid. Therefore, lower values of n_o and higher values of n_w are desired since this would lead to more piston-like displacement and higher displacement efficiency. Figure 7 (b) also shows the presence of inflection point for values of n_o higher than 3.

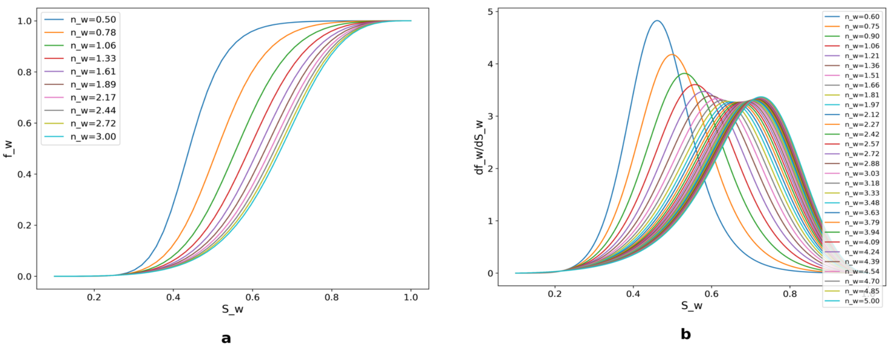


Figure 6 – Fractional flow (a) & fractional flow derivative (b) curves for different values of power-law index of displacing fluid

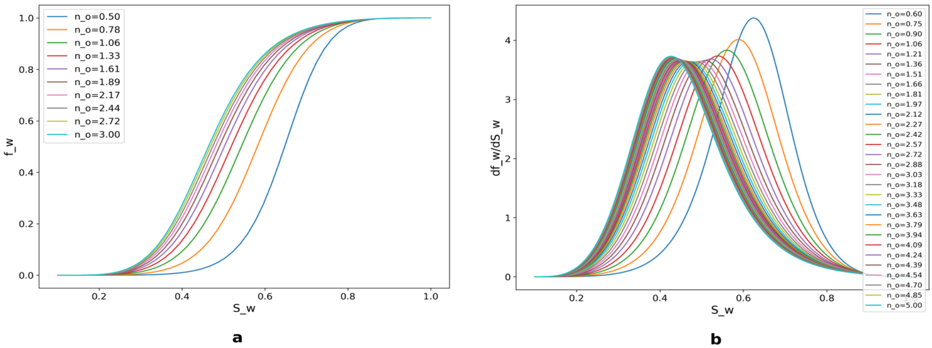


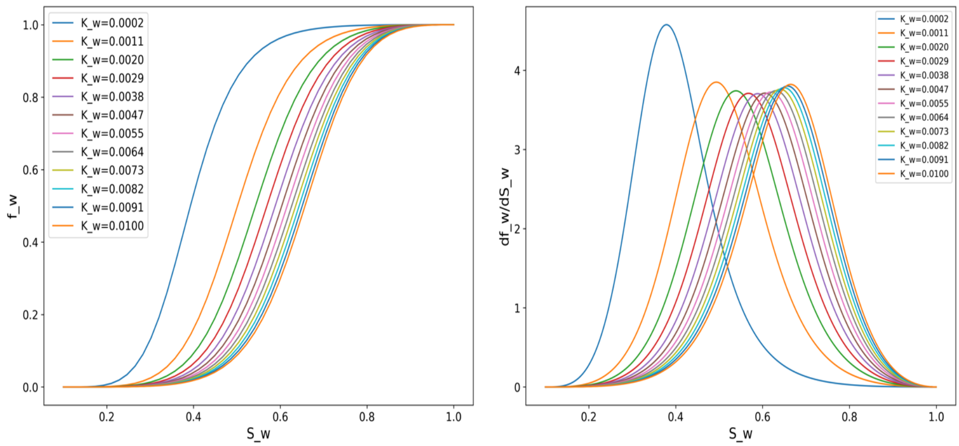
Figure 7 – Fractional flow (a) & fractional flow derivative (b) curves for different values of power-law index of displaced fluid

Figure 8 (a) shows the relationship between fractional flow and saturation for various values of consistency index of displacing fluid. It can be seen that the decrease of K_w increases the fractional flow of displacing fluid, which makes sense since the consistency index is an analogue of viscosity for non-Newtonian fluids. The most abrupt increase of fractional flow corresponds to the lower values of K_w.

Figure 8 (b) shows the relationship between derivative of fractional flow and saturation for different values of K_w. The decrease of K_w increases the derivative of fractional flow and decreases the displacing fluid saturation at the displacement front. An inflection point can be noticed for higher values of K_w.

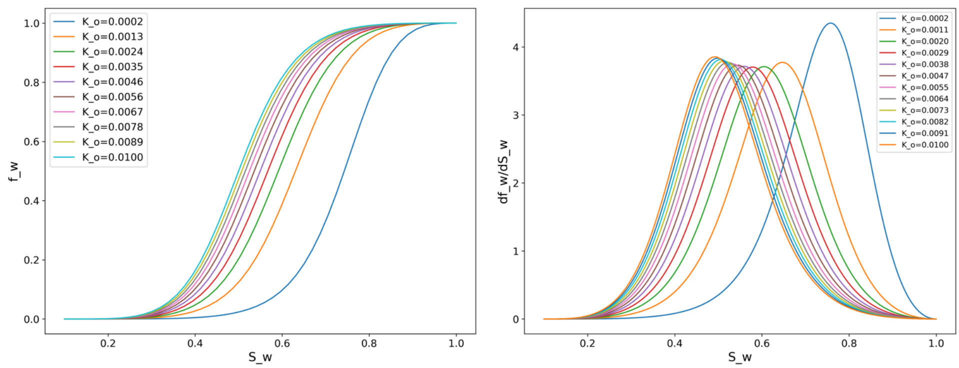
Figure 9 (a) shows the relationship between fractional flow and saturation for various values of consistency index of the displaced fluid. The decrease of K_o leads to the decrease of displacing fluid fractional flow. The most abrupt decrease of fractional flow can be observed at lower values of K_o.

Figure 9 (b) shows the relationship between fractional flow derivative and saturation for various values of consistency index of the displaced fluid. It can be concluded that the decrease of K_o increases the value of fractional flow derivative and also increases the displacing fluid saturation at the displacement front. An inflection point can be noticed here as well for higher values of K_o. Therefore, we can conclude that the increase of ratio K_w/K_o is desired since it makes the displacement more piston-like.



a b

Figure 8 – Fractional flow (a) & fractional flow derivative (b) curves for different values of consistency index of displacing fluid



a b

Figure 9 – Fractional flow (a) & fractional flow derivative (b) curves for different values of consistency index of displaced fluid

Figure 10 (a) shows the relationship between fractional flow and saturation for various values of initial pressure gradient of displacing fluid. We can see that the increase of G_{0_w} decreases the fractional flow of displacing fluid. The most abrupt decrease of fractional flow can be observed for higher values of G_{0_w}. When $G_{0_w} = 100000$ Pa/m, the fractional flow becomes equal to 0. This is related to the fact that the term $\Delta P - G_{0_w} L$ in equation (27) becomes equal to 0 at this value of G_{0_w}.

Figure 10 (b) shows the relationship between fractional flow derivative and saturation for various values of initial pressure gradient of displacing fluid. An inflection point is present here – when the values of G_{0_w} are low, the value of fractional flow derivative decreases with the increase of G_{0_w}. Then, after a certain inflection point, the fractional flow derivative starts to increase with the increase of

G_{0_w}. It can also be observed that the increase of G_{0_w} increases the displacing fluid saturation at the displacement front.

Figure 11 (a) shows the relationship between fractional flow and saturation for various values of initial pressure gradient of displaced fluid. It can be seen that the increase of G_{0_o} increases the value of fractional flow. The most abrupt increase of fractional flow can be noticed at higher values of G_{0_o}. When $G_{0_o} = 100000$ Pa/m the flow of displaced fluid stops, and the fractional flow becomes equal to 1.

Figure 11 (b) shows the relationship between the fractional flow derivative and saturation for various values of G_{0_o}. In this case an inflection point is absent and the value of derivative of fractional flow decreases with the decrease of G_{0_o}. The increase of G_{0_o} decreases the displacing fluid saturation at the displacement front.

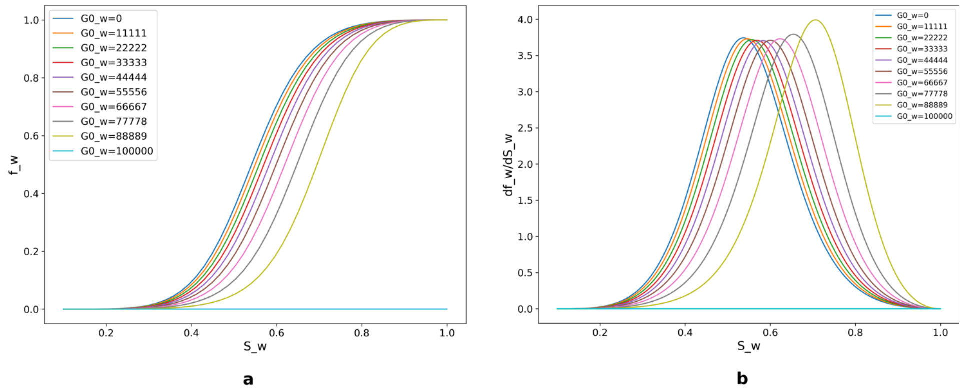


Figure 10 – Fractional flow (a) & fractional flow derivative (b) curves for different values of initial pressure gradient of displacing fluid

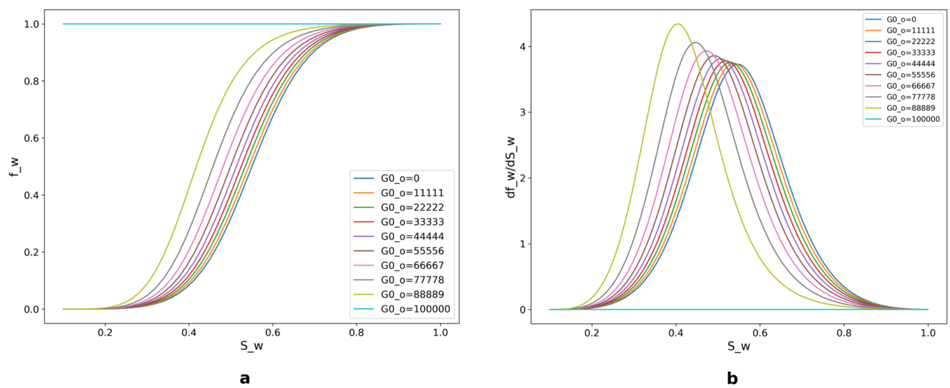


Figure 11 – Fractional flow (a) & fractional flow derivative (b) curves for different values of initial pressure gradient of displaced fluid

After a more detailed analysis it has been found that the presence of an inflection point on the graph of fractional flow derivative shown in cases above depends on the values of power-law indexes of displacing and displaced fluids. The graph shown on Figure 11 (b) corresponds to the power-law indexes $n_w = 0.95$ & $n_o = 1.05$. However, if we take for instance values $n_w = 1.5$ & $n_o = 0.8$, then we would get a completely different picture which is shown on Figure 12 below. It can be seen that for smaller values of G_{0_o}, the increase of G_{0_o} leads to the decrease of fractional flow derivative. However, after a certain inflection point, the increase of G_{0_o} starts increasing the derivative of fractional flow.

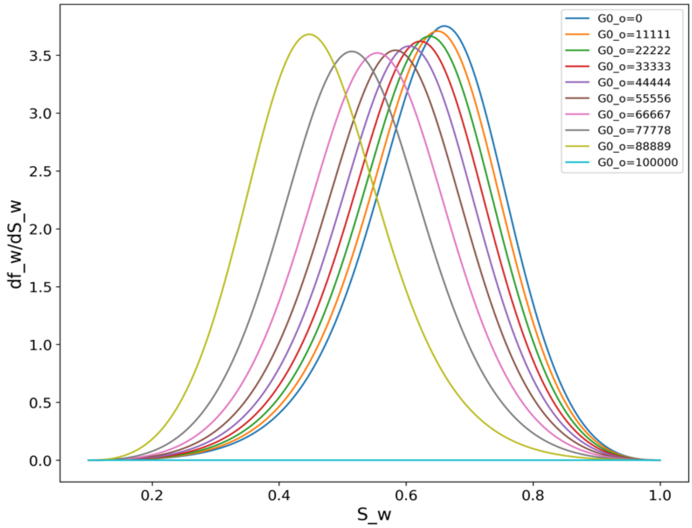


Figure 12 – Fractional flow derivative curves for different values of initial pressure gradient of displaced fluid for $n_w = 1.5$ & $n_o = 0.8$

The significance of constructing the fractional flow curve arises from the factors mentioned below:

• It allows to construct the relationship between the fractional flow derivative by saturation and saturation

• It allows to determine the saturation at any point in the reservoir at any time

• It allows to determine the frontal advance velocity for any saturation

• It allows to determine the breakthrough time of displacing fluid in the production well

• It allows to determine the sweep efficiency and displacement efficiency, and therefore, the oil recovery factor and the total oil production

• After the breakthrough of the displacing fluid into the production well, it allows to determine the flow rates of displaced and displacing fluids in the production well

Now let's illustrate the methods of determining the aforementioned parameters by using the Buckley-Leverett theory (Muhammad and Moataz, 2022; Ahmed, 2019). Figure 1 shows the displacing fluid saturation distribution in the reservoir at certain time before breakthrough to the production well. It can be seen that the maximum possible displacing fluid saturation $1 - S_{or}$ has the minimal front advance velocity. Let's assume that at any time the coordinate of saturation $1 - S_{or}$ is denoted by x_1. The coordinate x_2 corresponds to the displacement front at any time. A gradual decrease of saturation can be observed between the points x_1 and x_2. Right after the coordinate x_2 the saturation abruptly falls to the value S_{wc}. The saturation $\bar{\bar{S}}_w$ represents the average saturation in the reservoir between the coordinates 0 and x_2. In this case, the total volume of injected fluid (displacing fluid) is determined as:

$$W_i = x_2 A\phi\left(\bar{\bar{S}}_w - S_{wc}\right), \tag{32}$$

or:

$$\bar{\bar{S}}_w - S_{wc} = \frac{W_i}{x_2 A\phi}. \tag{33}$$

By using the definition of x_2 from equation (13) we get:

$$\bar{\bar{S}}_w - S_{wc} = \frac{1}{\frac{df_w}{dS_w}(S_{wf})}. \tag{34}$$

In the work (Civan, 2011) the following definition of average saturation was proposed:

$$\bar{\bar{S}}_w = \frac{(1 - S_{or})x_1 + \int_{x_1}^{x_2} S_w dx}{x_2} \tag{35}$$

Now we insert the definition of x from equation (13) into equation (35) and get the following:

$$\bar{\bar{S}}_w = \frac{(1 - S_{or})\frac{df_w}{dS_w}|_{1-S_{or}} + \int_{1-S_{or}}^{S_{wf}} S_w d\left(\frac{df_w}{dS_w}\right)}{\frac{df_w}{dS_w}|_{S_{wf}}}. \tag{36}$$

Using integration by parts we get:

$$udv = uv - vdu, \tag{37}$$

which results in the following:

$$\int_{1-S_{or}}^{S_{wf}} S_w d\left(\frac{df_w}{dS_w}\right) = \left[S_w \frac{df_w}{dS_w}\right]_{1-S_{or}}^{S_{wf}} - [f_w]_{1-S_{or}}^{S_{wf}}. \tag{38}$$

As a result, we get the following definition of average saturation:

$$\bar{\bar{S}}_w = S_{wf} + \frac{\left(1 - f_w|_{S_{wf}}\right)}{\frac{df_w}{dS_w}|_{S_{wf}}}. \quad (39)$$

By comparing equations (34) and (39) we get:

$$\frac{df_w}{dS_w}|_{S_{wf}} = \frac{1 - f_w|_{S_{wf}}}{\bar{\bar{S}}_w - S_{wf}} = \frac{1}{\bar{\bar{S}}_w - S_{wc}}. \quad (40)$$

Now let's refer to the fractional flow curve again. For the conditions mentioned in equation (40) to be satisfied, it is required for the conditions shown on Figure 13 below to be satisfied as well. On Figure 13 we can observe 2 right triangles – one small and one large. The large one has legs 1 and $\bar{\bar{S}}_w - S_{wc}$, while the small one has legs $1 - f_w|_{S_{wf}}$ and $\bar{\bar{S}}_w - S_{wf}$. The orange line is a tangent to the fractional flow curve from the point $S_w = S_{wc}$ to the point $f_w = 1$. Using this method, we can determine that in this case: $\bar{\bar{S}}_w = 0.753, S_{wf} = 0.685, f_w|_{S_{wf}} = 0.895.$ This method is known as the Welge method (Civan, 2011). It allows us to determine the saturation at the displacement front, average saturation in the reservoir and the fractional flow at the displacement front by only using the fractional flow curve.

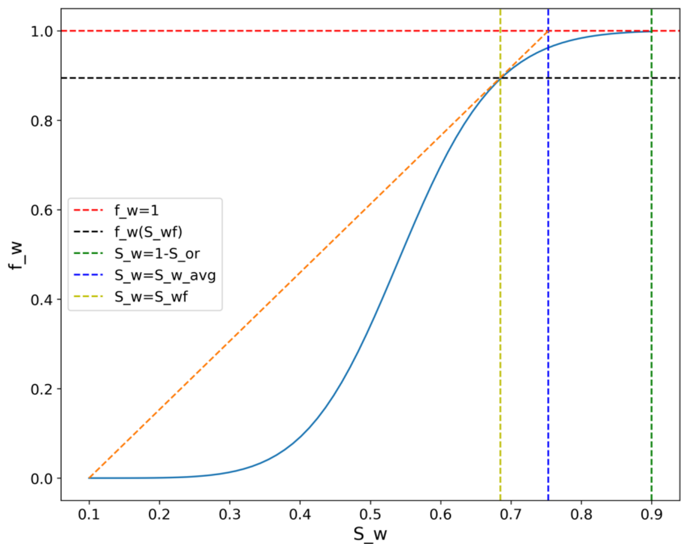


Figure 13 – Determination of saturation at the displacement front, average saturation in the reservoir and the fractional flow at the displacement front using the Welge method

After the breakthrough of the injected fluid, the relationship between the injected volume and fractional flow derivative, according to equation (13) becomes:

$$\frac{1}{\frac{df_w}{dS_w}(S_{we})} = \frac{W_i}{A\phi L} = W_{id}, \quad (41)$$

where S_{we} is the saturation of displacing fluid at the displacement front after breakthrough to the production well and obviously, the value of S_{we} increases with time. W_{id} is the dimensionless volume of injected fluid. After breakthrough, we assume that all saturations continue to move with the same frontal advance velocity as before breakthrough, and for this reason, each saturation sooner or later reaches the production well. The principal difference of production before breakthrough and after is that before breakthrough, the total volume of produced oil is equal to the total volume of injected fluid. Right after the breakthrough (when the displacement front reaches the production well) the fractional flow at the production well abruptly increases from 0 to the value that corresponds to the saturation at the displacement front. This effect is also usually noticed in practice (Ahmed, 2019). At the time of breakthrough, the following condition is satisfied:

$$N_{id_{bt}} = W_{id_{bt}} = Q_{id}t_{bt} = \bar{\bar{S}}_{w_{bt}} - S_{wc} = \frac{1}{\frac{df_w}{dS_w}(S_{wf})}, \quad (42)$$

where $N_{id_{bt}}$ is the dimensionless oil production at the time of breakthrough (which is defined as total oil production N_i divided by $A\phi L$), $W_{id_{bt}}$ is the dimensionless volume of injected fluid at the time of breakthrough, Q_{id} is total flow rate divided by $A\phi L$, and t_{bt} is the breakthrough time. From equation (42) we conclude that the breakthrough time is determined as:

$$t_{bt} = \frac{W_{id_{bt}}}{Q_{id}}. \quad (43)$$

After breakthrough, equation (39) is modified as follows:

$$\bar{\bar{S}}_w = S_{we} + \frac{\left(1 - f_w|_{S_{we}}\right)}{\frac{df_w}{dS_w}|_{S_{we}}}. \quad (44)$$

Using the definition of $\frac{df_w}{dS_w}|_{S_{we}}$ from equation (41) we can write:

$$\bar{\bar{S}}_w = S_{we} + \left(1 - f_w|_{S_{we}}\right)W_{id}. \quad (45)$$

We now consider that the total oil production is equal to the fluid volume that has been injected to the reservoir but has not been produced from it yet:

$$N_i = A\phi L(\bar{\bar{S}}_w - S_{wc}). \tag{46}$$

From equation (46) we conclude that the dimensionless oil production is defined as:

$$N_{id} = \bar{\bar{S}}_w - S_{wc}. \tag{47}$$

Then by deducting S_{wc} from the left and right part of equation (45) we get:

$$N_{id} = (S_{we} - S_{wc}) + (1 - f_w|_{S_{we}})W_{id}. \tag{48}$$

If we assume that the injection flow rate at the injection well is equal to $Q_t = 0.001\ m^3/s$, that the fluids are incompressible, and that the flow rate does not change with time, using the same parameter values from Table 1, we can show how the saturation changes in the reservoir with the change of x and t.

Figure 14 illustrates the distribution of saturation of displacing fluid in the reservoir at different times and how the displacement front advances with time (time is measured in days). The calculations show that in this case, the breakthrough time is approximately equal to 30 days. It can be seen that the displacement front reaches the production well quite quickly, after what the saturation at the production well continuously increases. It can also be noticed that the saturation at the production well grows slower with time.

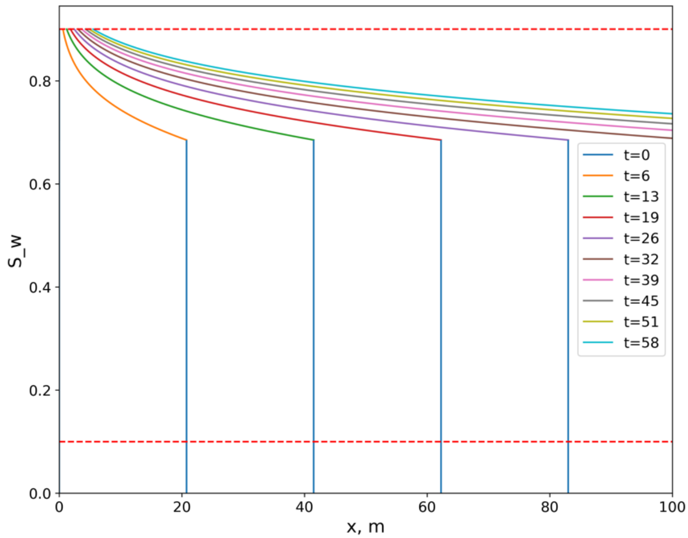


Figure 14 – Saturation profiles of displacing fluid in the reservoir at different times

Figure 15 (a) shows the relationship between total oil production and time. The red dashed line corresponds to the breakthrough time. It can be observed that before breakthrough, the total oil production abruptly increases. After breakthrough, the derivative of total oil production by time decreases, and continues to decrease with time. At the breakthrough time, the total oil production is equal to 2650 m^3. In this case, the initial oil reserves are equal to 3600 m^3 and the extractable oil reserves are 3200 m^3, if we assume that $S_{or} = 0.1$. Therefore, at the breakthrough time, approximately 83% of extractable reserves are produced. All the extractable reserves are produced after 1045 days which is significantly higher than the breakthrough time.

Figure 15 (b) shows how the average saturation changes with time in the flooded zone of the reservoir. Before breakthrough, the average saturation remains constant, and after breakthrough it gradually increases with time.

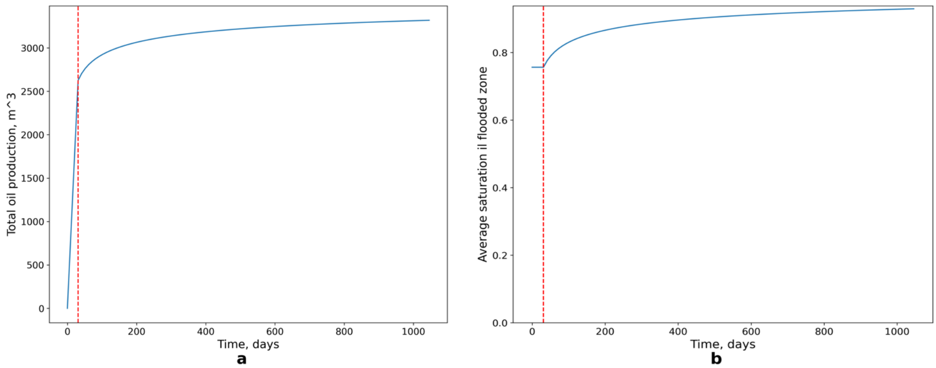


Figure 15 – Relationship between total oil production and time (a); Relationship between average saturation in flooded zone and time (b)

Figure 16 (a) shows how the flow rates of displaced and displacing fluid at the production well change with time. We can see that before breakthrough, the flow rate of displacing fluid is constant and equal to zero, meaning that only oil is produced from the production well. Before breakthrough, the flow rate of oil in the production well is equal to the injection rate which in this case we consider equal to $Q_t = 0.001$ м^3/с. After breakthrough the situation changes – the oil flow rate abruptly drops to approximately 0.0001 м^3/с, which is only 10% of Q_t. At the same time, the flow rate of displacing fluid becomes 0.0009 м^3/с which is 90% of Q_t. After breakthrough, the oil flow rate continuously decreases, while the displacing fluid flow rate continuously increases.

Figure 16 (b) shows how the sweep efficiency, displacement efficiency and oil recovery factor change with time. Sweep efficiency in this case stands for the fraction of length of reservoir covered by flooding. Displacement efficiency stands for the volumetric fraction of injected fluid within the volume of the reservoir affected by flooding. The oil recovery factor simply stands for the sweep efficiency multiplied by displacement efficiency. It can be seen that the sweep efficiency continuously increases until breakthrough and remains constant (equal to 1) after breakthrough. The displacement efficiency remains constant before breakthrough and starts continuously increasing after breakthrough. The oil recovery factor continuously increases during the whole time, however, the rate of increase drops significantly after breakthrough.

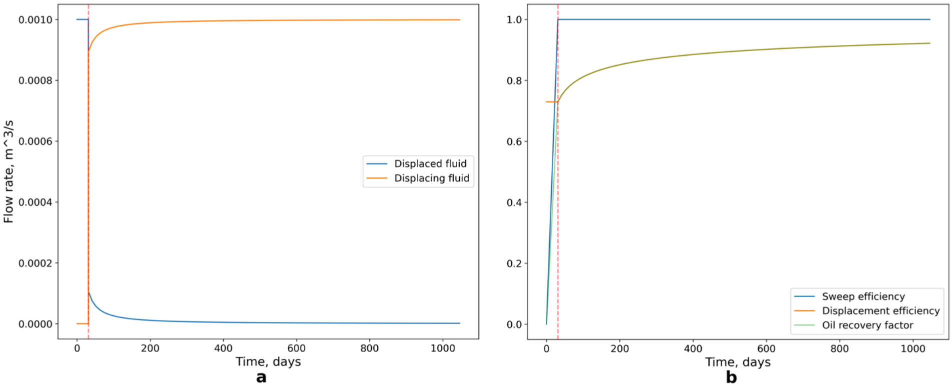


Figure 16 – Relationship between flow rates and time (a); Relationship between sweep efficiency, displacement efficiency, oil recovery factory and time (b)

Conclusion. The objective of this work was to extend the Buckley-Leverett theory in order to account for the immiscible displacement of one non-Newtonian Herschel-Bulkley fluid by another Herschel-Bulkley fluid. The proposed mathematical model is based on the modified Darcy law for Herschel-Bulkley fluids. Based on this, a modified fractional flow equation has been presented, and a fractional flow curve has been built by relating the fractional flow to saturation of the displacing fluid using the Corey correlation. It is assumed that the flow occurs under steady-state conditions, the reservoir is linear and homogeneous, capillary pressure is negligible, and the fluid is incompressible.

The impact of various non-Newtonian parameters of both the displacing and the displaced fluids on the relationship between fractional flow and displacing fluid saturation, and the relationship between the derivative of fractional flow by saturation and the saturation has been analyzed. It has been shown that the increase of power-law index, consistency index and initial pressure gradient of the displacing fluid decreases the fractional flow of the displacing fluid. On the other hand, the increase of power-law index, consistency index and initial pressure gradient of the displaced fluid increases the fractional flow of the displacing fluid.

The increase of power-law index, consistency index and initial pressure gradient of the displacing fluid lead to the increase of displacing fluid saturation at the displacement front, while increase of the same parameters of the displaced fluid decrease the saturation at the displacement front. The increase of power-law index and the consistency index for both the displacing and displaced fluid decreases the derivative of the fractional flow, while the increase of initial pressure gradient increases it. In many cases, inflection points can be spotted on the relationship curve between the aforementioned parameters and the derivative of fractional flow.

It has been shown that the presence of such inflection points depends on the ratio of power-law indexes of displaced and displacing fluids. By inflection points we mean that the relationship between the mentioned parameters and the fractional flow derivative deviates from the general trends shown above.

The main conclusion was that in order to achieve the maximum efficiency of displacement and prevent early breakthrough while having a low oil recovery factor, we need to have maximum values of power-law index, consistency index and initial pressure gradient for the displacing fluid, and minimum values of same parameters for the displaced fluid. This allows to achieve a piston-like displacement which leads to maximum displacement efficiency and oil recovery factor.

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