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Қ. И. Сәтпаев атындағы Қазақ ұлттық техникалық зерттеу университеті

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## ИЗВЕСТИЯ

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## NEWS

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## **SIMULATION OF THREE-PHASE TRANSFORMER OPERATIONAL CONDITIONS**

**Abstract.** With the purpose of improvement of power transformer current protection, it is necessary to know their behavior in various operating modes. This article is concerned with the mathematic simulation of three-phase power transformer operating modes. Building of mathematical model was carried out according to the equivalent circuit for double-wound transformer, that takes into account, apart from resistance and reactive resistance, mutual induction resistance and loads. The proposed mathematical model, written as a system of differential equation system, is universal, since it describes all the possible operating modes that occur during the operation of power double-wound intact transformer. The adequacy of the proposed mathematical model is confirmed by the results of experiments performed. Comparison of simulation results and experiments showed that the simulation error does not exceed the allowable values for relay protection.

**Key words:** transformer, mathematical model, protective relay, current protection.

The behavior analysis of a three-phase transformer protective relay is carried out under natural operational conditions and in an emergency operation mode. At the same time, considerable attention is paid to transient phenomena [1, 2].

As a rule, most of the time while in operation, the transformer runs in a steady state, and transients occur in it when switching from one steady state to another one, for example, when the voltage of the electrical network or load changes, and when the transformer is hooked up or during a short circuit. In turn, emergency transients in transformers occur during sudden short circuits in the windings, load or network [3, 4].

The process of energy conversion in transients and steady-state modes in the three-phase transformers is described to the fullest extent possible by a mathematical model with phase coordinates, the differential equations of which are built for phase voltages according to Kirchhoff's law [3-5]. However, such a model is not able to simulate these processes in transformers when voltage or load is unbalanced, since in this case the phase voltages are unknown. In addition, the calculated values of currents and voltages are obtained in reduced, but not in a natural form. To solve these problems is proposed as follows.

The circuit for the generation of differential equations of the mathematical model with the phase coordinates of a three-phase transformer is shown in figure 1.

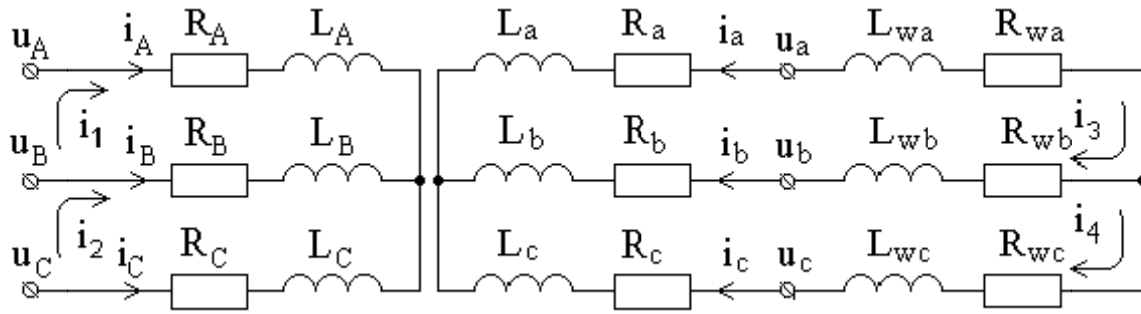


Figure 1 – Three-phase double-wound transformer circuit

In accordance with this circuit and [5], the operation of a three-phase transformer at a balanced voltage in the network and a balanced load is described by a system of differential equations

$$\left. \begin{aligned} u_A &= R_A i_A + d\psi_A/dt \\ u_B &= R_B i_B + d\psi_B/dt \\ u_C &= R_C i_C + d\psi_C/dt \\ 0 &= (R_a + R_{wa} + jL_{wa})i_a + d\psi_a/dt \\ 0 &= (R_b + R_{wb} + jL_{wb})i_b + d\psi_b/dt \\ 0 &= (R_c + R_{wc} + jL_{wc})i_c + d\psi_c/dt \end{aligned} \right\}, \quad (1)$$

where  $u_i, i_i$  – instantaneous phase voltages and currents in the primary and secondary windings ( $i = A, B, C$  and  $a, b, c$ );  $R_i$  and  $\psi_i$  – resistance and flux linkages of primary and secondary windings;  $Z_{wi} = R_{wi} + jL_{wi}$ ;  $R_{wi}$  and  $L_{wi}$  – resistance and load inductance of phase ( $i = a, b, c$ ).

The flux linkages of primary and secondary windings are expressed in terms of phase current and relevant inductance as follows:

$$\begin{aligned} \Psi_A &= i_A L_A + i_B L_{AB} + i_C L_{AC} + i_a L_{Aa} + i_b L_{Ab} + i_c L_{Ac}, \\ \Psi_B &= i_A L_{BA} + i_B L_B + i_C L_{BC} + i_a L_{Ba} + i_b L_{Bb} + i_c L_{Bc}, \\ \Psi_C &= i_A L_{CA} + i_B L_{CB} + i_C L_C + i_a L_{Ca} + i_b L_{Cb} + i_c L_{Cc}, \\ \Psi_a &= i_A L_{aA} + i_B L_{aB} + i_C L_{aC} + i_a (L_a + L_{wa}) + i_b L_{ab} + i_c L_{ac}, \\ \Psi_b &= i_A L_{bA} + i_B L_{bB} + i_C L_{bC} + i_a L_{ba} + i_b (L_b + L_{wb}) + i_c L_{bc}, \\ \Psi_c &= i_A L_{cA} + i_B L_{cB} + i_C L_{cC} + i_a L_{ca} + i_b L_{cb} + i_c (L_{Cc} + L_{wc}). \end{aligned} \quad (2)$$

It should be noted that in formulas (1) and (2) there used inductances associated only with the main magnetic flux in the transformer. In accordance with [6], the exact calculation of the inductance and inductive reactance of the windings in a three-phase transformer is rather complicated. However, if it is granted that the magnetic circuit of the transformer is not saturated, then they can be simplified as follows:

If the phase voltage  $U_l$ , the no-load current  $I_{nl}$  of the transformer and the resistance of the primary winding  $R_l$  are known, then the impedance and inductive reactance of the primary winding phase, as well as its inductance  $L_l$ , can be calculated [6] as follows:

$$Z_l = \dot{U}_l / \dot{I}_{nl}, \quad X_l = \sqrt{Z_l^2 - R_l^2}, \quad L_l = X_l / 2 \cdot \pi \cdot f_c, \quad (3)$$

where  $f_c$  - network frequency. It should be added that the phase voltage  $U_l$ , no-load current  $I_{nl}$  and resistance  $R_l$  of the primary winding are easy to determine experimentally.

Providing that the inductances of the transformer windings are proportional to the square of the number of turns, then the inductance of the secondary winding and the mutual induction of the primary and secondary windings are calculated as

$$L_2 = (w_2/w_1)^2 \cdot L_1 \quad \text{and} \quad L_{12} = ((w_1 \cdot w_2)/w_1)^2 \cdot L_1, \quad (4)$$

where  $w_1$  and  $w_2$  - number of turns in primary and secondary windings.

As a result, in mathematical expressions (1) and (2) the own inductances of the phases of the primary and secondary windings

$$L_A = L_B = L_C = L_1 \quad \text{and} \quad L_a = L_b = L_c = L_2. \quad (5)$$

In this setting mutual inductance between the phases of the windings

$$\begin{aligned} L_{AB} = L_{BC} = L_{CA} = L_1/2 \quad \text{and} \quad L_{ab} = L_{bc} = L_{ca} = L_2/2, \\ L_{Aa} = L_{Bb} = L_{Cc} = L_{Ac} = L_{Ba} = L_{Cb} = L_{Ab} = L_{Bc} = L_{Ca} = L_{12}. \end{aligned} \quad (6)$$

The resistance of the phases of the primary and secondary windings

$$R_A = R_B = R_C = R_1 \quad \text{and} \quad R_a = R_b = R_c = R_2, \quad (7)$$

where  $R_1$  and  $R_2$  - resistance of the phases of the primary and secondary windings of the transformer. They can be calculated by the known cross section and the length of the windings wire. In practice, the values of these resistances are easiest to obtain by measuring.

Since it is impossible to use the system of equations (1) when there is an unbalanced voltage in the network or an unbalanced load, in this case the differential equations of the mathematical model should be composed by the mesh-current method. A similar result can also be achieved by the rearrangement of the system of equations (1) in terms of subtracting the corresponding lines [5, 6] with the subsequent replacement of the phase currents by the loop currents. As a result of the corresponding lines

$$\left. \begin{aligned} u_{AB} &= i_A R_A - i_B R_B + d\psi_{AB}/dt \\ u_{BC} &= i_B R_B - i_C R_C + d\psi_{BC}/dt \\ 0 &= i_a (R_a + R_{wa}) - i_b (R_b + R_{wb}) + d\psi_{ab}/dt \\ 0 &= i_b (R_b + R_{wb}) - i_c (R_c + R_{wc}) + d\psi_{bc}/dt \end{aligned} \right\}, \quad (8)$$

where flux linkages

$$\begin{aligned} \psi_{AB} &= i_A (L_A - L_{AB}) + i_B (L_{AB} - L_B) + i_C (L_{AC} - L_{BC}) + \\ &+ i_a (L_{Aa} - L_{Ba}) + i_b (L_{Ab} - L_{Bb}) + i_c (L_{Ac} - L_{Bc}), \end{aligned} \quad (9)$$

$$\begin{aligned} \psi_{BC} &= i_A (L_{AB} - L_{AC}) + i_B (L_B - L_{BC}) + i_C (L_{BC} - L_C) + \\ &+ i_a (L_{Ba} - L_{Ca}) + i_b (L_{Bb} - L_{Cb}) + i_c (L_{Bc} - L_{Cc}), \end{aligned} \quad (10)$$

$$\begin{aligned} \psi_{ab} &= i_A (L_{aA} - L_{bA}) + i_B (L_{ab} - L_{bB}) + i_C (L_{ac} - L_{bc}) + \\ &+ i_a (L_a + L_{wa} - L_{ba}) + i_b (L_{ab} - L_b - L_{wb}) + i_c (L_{ac} - L_{bc}), \end{aligned} \quad (11)$$

$$\begin{aligned} \psi_{bc} &= i_A (L_{Ba} - L_{Ca}) + i_B (L_{bB} - L_{cB}) + i_C (L_{bc} - L_{cC}) + \\ &+ i_a (L_{ba} - L_{ca}) + i_b (L_b + L_{wb} - L_{cb}) + i_c (L_{bc} - L_c - L_{wc}). \end{aligned} \quad (12)$$



The replacement of the phase currents by the loop currents is carried out by applying the equations

$$\dot{i}_A = \dot{i}_1, \dot{i}_B = \dot{i}_2 - \dot{i}_1, \dot{i}_C = -\dot{i}_2, \dot{i}_a = \dot{i}_3, \dot{i}_b = \dot{i}_4 - \dot{i}_3, \dot{i}_c = -\dot{i}_4, \quad (13)$$

which are composed according to the circuit in figure 1.

As a result, after substituting formulas (13) into the system of equations (8) and rearranging it, it is possible to obtain a new system in terms of

$$\left. \begin{aligned} u_{AB} &= \dot{i}_1(R_A + R_B) - \dot{i}_2 R_B + d\psi_1/dt \\ u_{BC} &= (\dot{i}_2 - \dot{i}_1)R_B + \dot{i}_2 R_C + d\psi_2/dt \\ 0 &= \dot{i}_3(R_a + R_b + R_{wa} + R_{wb}) - \dot{i}_4(R_b + R_{wb}) + d\psi_3/dt \\ 0 &= (\dot{i}_4 - \dot{i}_3)(R_b + R_{wb}) + \dot{i}_4(R_c + R_{wc}) + d\psi_4/dt \end{aligned} \right\}. \quad (14)$$

In this system of flux linkage of the circuits

$$\begin{aligned} \psi_1 &= \dot{i}_1 L_{11} + \dot{i}_2 L_{12} + \dot{i}_3 L_{13} + \dot{i}_4 L_{14}, \\ \psi_2 &= \dot{i}_1 L_{21} + \dot{i}_2 L_{22} + \dot{i}_3 L_{23} + \dot{i}_4 L_{24}, \\ \psi_3 &= \dot{i}_1 L_{31} + \dot{i}_2 L_{32} + \dot{i}_3 L_{33} + \dot{i}_4 L_{34}, \\ \psi_4 &= \dot{i}_1 L_{41} + \dot{i}_2 L_{42} + \dot{i}_3 L_{43} + \dot{i}_4 L_{44}. \end{aligned} \quad (15)$$

In this case internal inductances of the circuit

$$\begin{aligned} L_{11} &= L_A - 2 \cdot L_{AB} + L_B, \\ L_{22} &= L_B - 2 \cdot L_{BC} + L_C, \\ L_{33} &= L_a + L_{wa} - 2 \cdot L_{ab} + L_b + L_{wb}, \\ L_{44} &= L_b + L_{wb} - 2 \cdot L_{bc} + L_c + L_{wc}, \end{aligned} \quad (16)$$

and mutual inductances of these circuits, consequently

$$\begin{aligned} L_{12} = L_{21} &= L_{AB} - L_{AC} - L_B + L_{BC}, \quad L_{23} = L_{32} = L_{Ba} - L_{Bb} - L_{Ca} + L_{Cb}, \\ L_{13} = L_{31} &= L_{Aa} - L_{Ab} - L_{Ba} + L_{Bb}, \quad L_{24} = L_{42} = L_{Bb} - L_{Bc} - L_{Cb} + L_{Cc}, \\ L_{14} = L_{41} &= L_{Ab} - L_{Ac} - L_{Bb} + L_{Bc}, \quad L_{34} = L_{43} = L_{ab} - L_{ac} - L_b + L_{bc}. \end{aligned} \quad (17)$$

Such mathematical model makes it possible to simulate the processes in a transformer at both unbalanced supply voltage and unbalanced load. In this case, the currents in all windings of the transformer are obtained in their natural form, which, in turn, greatly facilitates the analysis of processes in any transients.

The switching from one steady state to another one involving system of equations (14) is simulated as follows. At the initial instant of the transformer transient mode, the currents  $\dot{i}_1 \div \dot{i}_4$  in the primary and secondary windings are taken to be equal to these currents as they are at the time when the previous mode ends. For example, when the transformer is hooked up into the network, the currents  $\dot{i}_1 \div \dot{i}_4$  in the primary and secondary windings are taken to be equal to zero. The time of the process under study is divided into time intervals with duration  $\Delta t$  and it is considered that the currents  $\dot{i}_1 \div \dot{i}_4$  within each time interval are constant. When passing from time interval  $q$  to time interval  $q+1$ , these values are calculated using the formulas below.

In a transient mode, the current in circuit  $i$  [7] is calculated as the sum of the current  $i_{pi}$  of the periodic (forced) and aperiodic (free) components, i.e.

$$\dot{i}_i = \dot{i}_{pi} + \dot{i}_{ai}. \quad (18)$$

The periodic component of current  $i$  in the time interval  $q$  is sought as a particular solution of the system of non-homogeneous equations obtained from (14). In this system, voltages are considered to be sinusoidal, and differentiation operator  $d/dt$  is replaced by  $j\omega$  [7]. As a result, system (14) is transformed to

$$\left. \begin{aligned} \dot{U}_{AB} &= [(R_A + R_B) + jX_{11}] \dot{i}_1 + (-R_B + jX_{12}) \dot{i}_2 + jX_{13} \dot{i}_3 + jX_{14} \dot{i}_4 \\ \dot{U}_{BC} &= (-R_B + jX_{21}) \dot{i}_1 + [(R_B + R_C) + jX_{22}] \dot{i}_2 + jX_{23} \dot{i}_3 + jX_{24} \dot{i}_4 \\ 0 &= jX_{31} \dot{i}_1 + jX_{32} \dot{i}_2 + [(R_a + R_b + R_{wa} + R_{wb}) + jX_{33}] \dot{i}_3 + [(-R_b + R_{wb}) + jX_{34}] \dot{i}_4 \\ 0 &= jX_{41} \dot{i}_1 + jX_{42} \dot{i}_2 + [(-R_b + R_{wb}) + jX_{43}] \dot{i}_3 + [(R_c + R_b + R_{wc} + R_{wb}) + jX_{44}] \dot{i}_4 \end{aligned} \right\}, \quad (18)$$

where inductances are calculated as  $X_{vw} = \omega L_{vw}$ , and  $v$  and  $w$  take on a value from one to four.

The aperiodic component of the current  $i_{ai}$  is sought as a full solution of a system of homogeneous differential equations. This system is also derived from (14), taking  $u_{AB} = 0$  and  $u_{BC} = 0$ . In the numerical solution of this system  $dt$  and  $di$  in it are replaced by  $\Delta t$  and  $\Delta i$ . Then the components of the resistance voltage drop are transposed to the left-hand side of the equations. As a result, the change in currents  $\Delta i$  in the circuit elements over the time interval is calculated by solving the system of equations

$$\left. \begin{aligned} [R_B i_2 - (R_A + R_B) i_1] \Delta t &= L_{11} \Delta i_1 + L_{12} \Delta i_2 + L_{13} \Delta i_3 + L_{14} \Delta i_{14} \\ [R_B i_1 - (R_B + R_C) i_2] \Delta t &= L_{21} \Delta i_1 + L_{22} \Delta i_2 + L_{23} \Delta i_3 + L_{24} \Delta i_{14} \\ [R_b i_4 - (R_a + R_b) i_3] \Delta t &= L_{31} \Delta i_1 + L_{32} \Delta i_2 + L_{33} \Delta i_3 + L_{34} \Delta i_{14} \\ [R_b i_3 - (R_b + R_c) i_4] \Delta t &= L_{41} \Delta i_1 + L_{42} \Delta i_2 + L_{43} \Delta i_3 + L_{44} \Delta i_{14} \end{aligned} \right\} \quad (19)$$

Since at the initial instant of the transformer transient mode, the currents in the circuits are taken to be equal to these currents as they are at the time when the previous mode ends, the aperiodic component of the current in them at the time of load-break switch contact closure is taken to be equal to the periodic component but with the opposite sign. And its value in circuit  $i$  within the interval  $q+1$  is calculated as

$$\dot{i}_{ai,q+1} = \dot{i}_{ai,q} + \Delta_{ai,q}. \quad (20)$$

Thus, the obtained mathematical model allows simulating almost all steady-state and transient processes in an intact three-phase transformer.

This mathematical model verification was carried out using an experimental transformer of TSZI-6 type (three-phase dry-type self-cooled transformer), in which the transformed secondary winding had 12 turns of copper wire with a cross section  $2,5 \text{ mm}^2$ . Parameters of this transformer are shown in table. Rheostats with a resistance of 6.7 Ohm were used as the load.

Parameters of an experimental transformer of TSZI-6 type

Parameters of transformer of TSZI-6 type	Notation	Value
Phase voltage, V	$U_1$	230
No-load current, A	$I_{nl}$	0,31
Number of turns in primary/secondary windings	$w_1/w_2$	447/12
Resistance of primary/secondary windings, Ohm	$R_1/R_2$	1,26/0,06
Load resistance, Ohm	$R_w$	0,165-6,7
Load inductive reactance, Ohm	$X_w$	0

The results of simulation and experiment in the form of currents in the windings of the experimental transformer at a balanced voltage in the network and a balanced load depending on its value are shown in figure 2, where the first are represented by lines and the other one - by dots. To simulate the processes on a PC, the application package MATLAB 6.1 was used.

Correlation of the results of calculations and experiments shows that the modeling error in the whole range of allowable loads does not exceed 10-12%.

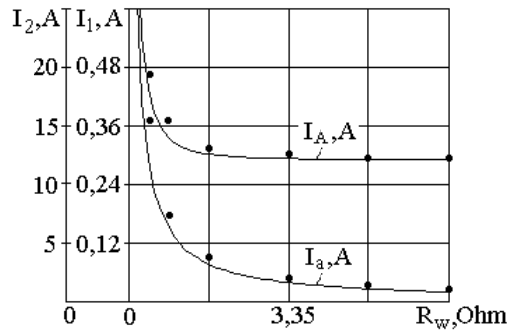


Figure 2 – The results of simulation and experiment of currents in the windings in the transformer of TSZI-6 type at a balanced supply voltage and load

Figures 3, a and 3, b show the results of the simulation of currents in the windings in the transformer of TSZI-6 type at an unbalanced supply voltage and an unbalanced load. The relationships in figure 3, a are obtained at  $U_{AB} = 0,8U_1$ ,  $U_{BC} = U_1$  and  $U_{CA} = U_1$ , where  $U_1$  is the line voltage of the network. In turn, the relationships in figure 3, b were obtained at an unbalanced load specified as  $R_{wa} = 0,7R_w$ ,  $R_{wb} = 0,7R_w$  and  $R_{wc} = 0,7R_w$ .

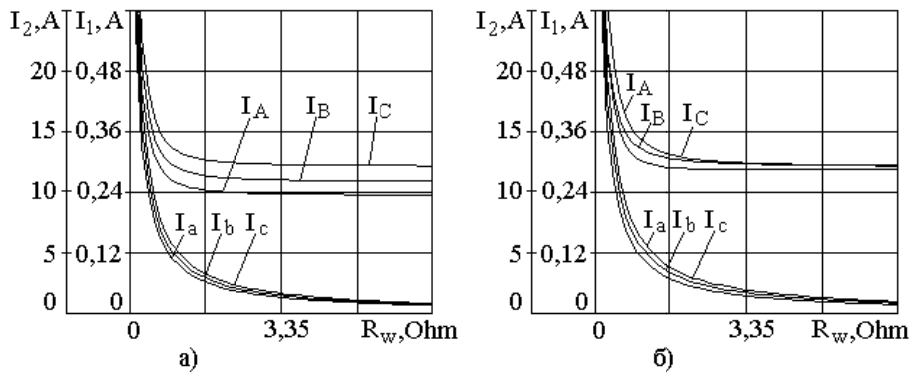


Figure 3 – The results of the simulation of currents in the windings in the transformer of TSZI-6 type when there is unbalance of supply voltage and load

The capabilities of the mathematical model, when simulating transients, were carried out using, as an example, a three-phase short circuit at the secondary winding end at  $R_w = R_d = 0,165$  Ohm, where  $R_d$  is the arc resistance. The simulation results in the form of currents in primary and secondary windings are shown in figure 4. Correlation of the results of calculations and the experiment shows that the modeling error does not exceed 10%.

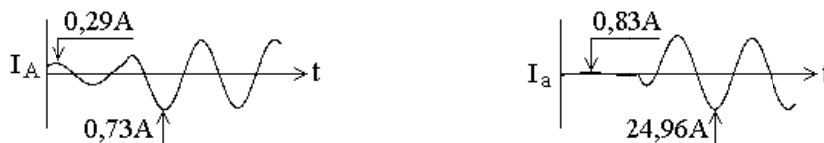


Figure 4 – The results of the simulation of currents in the phase A windings in the transformer of TSZI-6 type in event of a three-phase short circuit at the secondary winding end

Thus, the proposed mathematical model of a three-phase transformer makes it possible to simulate the currents magnitudes in its windings in both steady-state mode and transient mode with accuracy acceptable for protective relay.

The process of energy conversion in the three-phase transformers in event of unbalance of supply voltage and load in transients and steady-state modes is described to the fullest extent possible by a mathematical model, the differential equations of which are composed using the mesh-current method for line voltage.

The modeling error of currents in the transformer windings in the whole range of allowable loads when there is unbalance of supply voltage and load, as well as in transients applying this model does not exceed 10-12%.

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### **ҮШ ФАЗАЛЫ КҮШ ТРАНСФОРМАТОРЫНЫҢ ЖҰМЫС РЕЖІМДЕРІН МАТЕМАТИКАЛЫҚ ҮЛГІЛЕУІ**

**Аннотация.** Күш трансформаторларының ток қорғанысын жетілдіру үшін олардың түрлі пайдаланушылық режимдердегі әрекеттерін білу қажет. Осы мақала үш фазалы күш трансформаторының жұмыс режимдерін математикалық үлгілеуге арналған. Математикалық үлгіні құру екі орамалы трансформаторды алмастыру сызбасы бойынша жүзеге асырылған, онда белсенді және реактивті кедергілерден басқа өзара индукция мен жүктеу кедергісі ескерілген. Дифференциалдық теңдеулер жүйесі түрінде жазылған ұсынылған математикалық үлгі әмбебап, өйткені ол зақымдалмаған екі орамды күш трансформаторын пайдаланған кезде туындайтын барлық мүмкін жұмыс режимдерін сипаттайды. Ұсынылған математикалық үлгінің баламалылығы жүргізілген тәжірибелердің нәтижелерімен расталған. Үлгілеу және тәжірибелердің нәтижелерін салыстыру үлгілеудің ауытқуы релелік қорғаныс үшін мүмкін мәндерден артпайтынын көрсетеді.

**Түйін сөздер:** трансформатор, математикалық үлгі, релелік қорғаныс, ток қорғанысы.

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### **МОДЕЛИРОВАНИЕ ЭКСПЛУАТАЦИОННЫХ РЕЖИМОВ РАБОТЫ ТРЕХФАЗНОГО ТРАНСФОРМАТОРА**

**Аннотация.** Для совершенствования токовых защит силовых трансформаторов необходимо знать поведение их в различных эксплуатационных режимах. Данная статья посвящена математическому моделированию режимов работы трехфазного силового трансформатора. Построение математической модели осуществлено по схеме замещения двухобмоточного трансформатора, в которой учитывались, кроме активных и реактивных сопротивлений, сопротивления взаимной индукции и нагрузки. Предлагаемая математическая модель, записанная в виде системы дифференциальных уравнений, универсальна, так как она описывает все возможные режимы работы, которые возникают при эксплуатации силового двухобмоточного неповрежденного трансформатора. Адекватность предложенной математической модели подтверждена результатами проведенных экспериментов. Сопоставление результатов моделирования и экспериментов показало, что погрешность моделирования не превышает допустимых значений для релейной защиты.

**Ключевые слова:** трансформатор, математическая модель, релейная защита, токовая защита.

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